Price Signaling and the Strategic Benefits of Price Rigidities*

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Abstract

We analyze trade between a price setting party (seller) who has private information about the quality of a good and a price taker (buyer) who may also have private information. Differently from most of the literature, we focus on the case in which, under full information, it would be inefficient to trade goods of poor quality. We show that there is a unique equilibrium outcome passing Cho and Kreps (1987) "Never a Weak Best Response". The refined outcome is always characterized by absence of trade, although trade would be mutually beneficial in some state of nature. This occurs: 1. Even if the price taker has more precise information than the price setting party, and 2. Even when the information received by both parties is almost perfect. The model thus implies that signaling through prices may exacerbate the effect of adverse selection rather than mitigate it. The price setting party would always benefit from committing to prices that do not reveal her information. We develop this intuition by analyzing the strategic advantages generated by price rigidities. Possible applications to professional bodies and compensation policies are discussed.

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1 Introduction

Since Akerlof (1970), a number of studies have argued that posted prices may be used to separate high quality goods from lemons.\(^1\) The adverse selection problem would be thus mitigated by the use of prices as a signaling device. A standard assumption in this literature is that, under full information, there are gains from trading any of the qualities of the good potentially offered by the sellers. This assumption is certainly valid in many applications. It is also motivated by the need for a clear benchmark when assessing the inefficiencies induced by asymmetric information. However, for certain goods and services, the idea that there are positive gains from trade irrespectively of quality sounds far less appealing. It is conceivable that the value to a patient of a diagnosis made by a skilled doctor greatly exceed the doctor’s cost of making the diagnosis. Yet, the value to the same patient of a diagnosis made by a quack might be zero or negative even though the quack incurs a cost when making the diagnosis. Similarly, the value to a client of the assistance of an incompetent lawyer could well be lower than the cost that the lawyer incurs to study the court case, while the opposite is probably true in the case of a talented lawyer.

This paper considers a model where, for some qualities of a given good or service, trade is socially inefficient. We show how the presence of these qualities dramatically impacts upon the effect of signaling through prices. We also argue that, when these qualities are present, price rigidities may afford strategic benefits to the price setting party by reducing the scope for price signaling.

We consider the problem of trade between a price setting party and a price taker, who both have private information. More precisely, both parties observe a private signal about the quality of the good. Consistent with most applications, we will mainly discuss the case in which the price setting party is the seller (but we also consider the symmetric case of a price setting buyer). The key feature of our setup is that it is inefficient to trade goods of poor quality – namely, the seller’s valuation for the lowest

quality is higher than the buyer’s valuation.\textsuperscript{2} Our analysis shows that whenever the price setting party has private information and the price taker’s information is not perfect there is a unique equilibrium outcome which survives Cho and Kreps (1987) “Never a Weak Best Response” (NWBR). Remarkably, in the refined outcome, trade always collapses.

The result is general to the extent that it holds independently of: a. The precision of the information available to the parties, b. The relative precision of the information of one party vis-à-vis the other. Trade collapses even if the buyer’s information is extremely precise or if it is extremely precise relative to the seller’s information. It is sufficient that the buyer is imperfectly informed and that the seller has some information not directly available to the buyer but potentially revealable through the price choice.\textsuperscript{3}

The rationale behind market breakdown differs substantially from the Akerlof (1970) model where all agents are price takers. In that model the market breaks down because adverse selection exerts a self-reinforcing downward pressure on the market price. In our model, trade collapses as a result of the seller’s attempts to signal quality through the price. This generates an \textit{upward} pressure on price.\textsuperscript{4} Hence, price setting plays a characterizing role in our model: market breakdown is a consequence of strategic pricing decisions. An important implication is that market breakdown occurs even if agents’ information is precise enough to guarantee trade if both parties were price takers. Far from alleviating the adverse selection problem, the use of prices as signals might exacerbate it.

The intuition behind the result stems from two observations. The first is that full separation through prices is never incentive compatible. The seller may have incentive to reveal her information through prices only if the buyer buys with a higher probability at low prices than at high prices. However, this relationship between price

\textsuperscript{2}Symmetrically, when the buyer sets the price, we consider the case in which, for the highest quality, the buyer’s valuation is exceeded by the seller’s valuation.

\textsuperscript{3}The market breakdown outcome is also quite robust. For instance, it is not affected by perturbations of the the payoffs as those used by Ellingsen (1997).

\textsuperscript{4}When the price is set by the buyer, this results in a \textit{downward} pressure on price. Notice however that, in the case of an informed buyer, standard adverse selection exerts an \textit{upward} pressure on prices.
and probability to trade breaks down when the realization of the seller’s signal is sufficiently bad, since gains from trade are negative for sufficiently low quality. Prices which reveal extremely bad realizations and induce the buyer to buy would be always below the minimum price at which the seller is willing to sell.

The second observation relates to the signaling role of prices out of the equilibrium path. It is well known that beliefs satisfying NWBR allow for this type of signaling. The effect of these “out of equilibrium” signals is to destroy any type of pooling that may sustain trade in equilibrium. This result is standard in models of price signaling – see, for instance, Bagwell (1991) and Ellingsen (1997). To gather intuition, suppose that the seller announces the same equilibrium price for two different realizations of her signal, say good and bad. In the candidate equilibrium, the seller’s profits from trading are lower when her signal is good than when it is bad. The incentive to deviate from the equilibrium price and announce a higher price is thus affected by the signal realization. According to NWBR, buyer’s beliefs should take this into account when observing a deviation. This in turn ensures that, upon receiving a good signal, the seller wants to deviate to a higher price. Any equilibrium in which trade is sustained by pooling is accordingly not robust.

The driving force of our analysis is that the signaling role of prices destroys trade, even though trade would be the efficient outcome in some state of nature. As a result, the price setting party would be always better off by committing ex-ante to strategies that are not contingent on her private information. We use this intuition to shed a new light on the role of price rigidities. In our setup, the price setting party can achieve a positive amount of trade by making her own pricing decisions less flexible.

Economists have traditionally viewed price rigidities as obstacles to mutually beneficial exchanges. How can price rigidities possibly promote trade? In short, our idea is that rigidities may reduce seller’s use of price as a signaling device. In turn, this allows the price setting party to realize higher profits in expectation. For example, a doctor could benefit if her professional association were to force her to set fees at a particular level, under threat of disciplinary action. Since the professional association
dictates an homogeneous fee to a large number of doctors, its fee does not reveal any information about the ability of a specific doctor.

To analyze the role of price rigidities, we consider an extension of the basic model in which trading at prices different from an ex-ante pre-determined price involves some cost. Such a cost can be interpreted as a measure of the degree of price rigidity. We show that whenever prices are not perfectly flexible (the cost is positive), there is a refined equilibrium in which the expected amount of trade is positive. In this equilibrium, prices do not reveal the seller’s information. Remarkably, the expected amount of trade as well as the expected surplus that is generated may increase in the degree of price rigidity. Our results thus imply that the price setting party may have little incentive to invest in technologies that enhance (or to promote institutional changes that favor) price flexibility. In recent years, a mix of deregulation and technological progress has removed many sources of rigidities in markets such as air travel, car rental, and internet retailing. Clearly, there are limits to the degree of price flexibility that can be achieved. Our results suggest that these limits might be dictated not only by technological or institutional constraints, but also by strategic considerations.

The paper is organized as follows. Section two briefly reviews the existing literature. Section three considers the case of a price-setting seller. Section four extends the results to the case of a price-setting buyer. Section five illustrates the benefits of price rigidities and discusses two possible applications: professional bodies and the "task attractiveness" model. Section six concludes.

2 Related Literature

This paper mainly contributes to two related strands of the literature. The first is the literature on the signaling role of prices in markets with imperfect information. The second is the literature on the effect of information on trade.

Within the first strand, important contributions including Milgrom and Roberts (1986), Laffont and Maskin (1987), Bagwell and Riordan (1991) Bagwell (1991), Overgard (1993) and Ellingsen (1997) have focused on the case of monopoly. In particular,
Bagwell (1991) finds that, with a downward sloping demand, the only equilibrium that satisfies the Intuitive Criterion (Cho and Kreps 1987) is a separating equilibrium in which the high quality is traded at a higher price but the amount sold is lower than the amount sold when the quality is low. Ellingsen (1997), who uses a setup more directly comparable with the one developed here, considers a model with one seller and one buyer with inelastic demand. He finds that there is a unique equilibrium which survives D1 (Cho and Kreps 1987) and is fully separating. Finally, Voorneveld and Weibull (2005) characterize all equilibria in a model with a perfectly informed seller and a buyer who observes a private signal. All these models assume that a positive amount of trade is efficient independently of the monopolist's quality. In other words, under full information, there are positive gains from trading each of the qualities of the good. Bester and Ritzberger (2001) consider a model in which buyers can choose to collect information at some cost. As in our model, prices cannot fully reveal the seller’s information in equilibrium. In their case, the result is driven by endogenous information acquisition. In ours, the result stems from the assumption of negative gains from trading a particular quality. It is also worth noting that none of the mentioned papers considers the case of a seller with imperfect information or the case of a price setting buyer.\footnote{Wilson (1980) considers the case of price setting buyers in a market populated by many agents. However, differently from our setup, the price taking sellers are perfectly informed.}

Within the second strand of literature, Levin (2001) considers a model in which both buyer and seller are price taker. He finds that the relationship between information asymmetries and trade is not monotonic. Kessler (2001), in a competitive market, finds similar results by varying the fraction of informed sellers. We show, in a price setting environment, that there is a discontinuity of the refined equilibrium in the precision of information available. When the price setting party has perfect information, the amount of trade drops from the full information level to zero as soon as the price taker’s information becomes noisy.\footnote{Mailath et al (1993) use a similar discontinuity argument to criticize the use of refinements based on forward induction. While we do not have strong views on the issue, the use of a forward induction refinement makes our results more directly comparable with the existing literature on the signaling role of prices (see for example Milgrom and Roberts (1986), Bagwell and Riordan (1991) Bagwell
The rationale for our result on price rigidities is similar to that given by Jullien and Mariotti (2005) for auctions with informed sellers. They consider second price auctions in which the seller’s information can be revealed through the reserve price. As they show, the introduction of an uninformed intermediary may increase the ex-ante probability of trade as well as expected surplus.

Finally, some of our findings echo results obtained within the principal-agent framework. In an informed-principal model, Ottaviani and Prat (2001) show that the principal always benefits from committing to directly reveal the information that can be guessed through the contract he offers and destroying the rest. In our setup, in which direct revelation is not possible, the monopolist is always better off by committing to a “contract” that does not reveal any information. Cremer (1995), in a model with moral hazard and adverse selection, shows that the principal may want to commit not to acquire information about the agent’s type in order to be able to make credible threats. The rationale for our results is different since we do not consider moral hazard. Finally, another work that is related to ours is Benabou and Tirole (2003) model of intrinsic and extrinsic incentives. In section 5, we discuss how their theory provides an alternative interpretation for our results.

3 The Price-Setting Seller Model

A seller ($S$) is endowed with one unit of a good. The quality $q$ of the good can be either high ($H$) or low ($L$). A buyer ($B$) has inelastic demand and consumes either one unit or nothing. The timing of the game is as follows. At stage 0 Nature draws $q \in \{H, L\}$ from a Bernoulli distribution with $\Pr(q = H) = \lambda$ and $\Pr(q = L) = 1 - \lambda$. At stage 1, either $S$ or both $S$ and $B$ exogenously receive some private information about the quality of the good (see below). At stage 2, $S$ announces a price $p$ at which she is willing to trade. The price announced by $S$ is assumed to be a take-it-or-leave-it offer. At stage 3, $B$ observes $p$ and chooses whether to buy or not. Finally, at stage 4, payoffs are realized.

(1991), and Ellingsen (1997)).
With respect to the information available to $S$ and $B$, we first consider the limiting case in which $S$ is perfectly informed while $B$ only relies on prior information and on the information conveyed by the price. We then turn attention to the general case in which both $S$ and $B$ receive a private signal, $s_S$ and $s_B$ respectively.

Seller $S$ values a high quality good $v_H$ and a low quality good $v_L < v_H$. Alternatively, $v_q$, $q = L, H$, can be interpreted as the cost of producing a good of quality $q$ (provided that production occurs conditional on trade). $B$ values a high quality good $u_H$ and a low quality good $u_L < u_H$. We assume $u_H > v_H$ and $u_L < v_L$ so that there is a potential gain from trading the high quality and a potential loss from trading the low quality. The last assumption is an aspect in which this model departs from the existing literature. As discussed in the introduction, one can think of several examples in which this assumption is appropriate, including professional advise by physicians, engineers, psychologists, etc.

The equilibrium concept we adopt is Perfect Bayesian Equilibrium (PBE). We give the definition for the general case. Denote with $\mu(q, s_S|p, s_B)$ the belief function giving $B$’s probability assessment that quality is $q \in \{H, L\}$ and that $S$ has observed a signal realization $s_S$ given $p$ and $s_B$. A PBE for the general case is a strategy profile for $S$ and $B$ and a belief function $\mu^*(q, s_S|p, s_B)$ which satisfy the usual conditions: 1) $S$’s best reply, 2) $B$’s best reply, 3) consistency of $\mu^*(q, s_S|p, s_B)$ for all $p$ that are announced with positive probability in equilibrium. In order to avoid the common “unsent message” problem, we refine the PBE concept with Cho and Kreps (1987) version of “Never a Weak Best Response” (NWBR). Intuitively, for any $p$ that is announced with probability zero in equilibrium, if the set of $B$’s best responses for which a seller observing $s_S$ weakly benefits from announcing $p$ (relative to her equilibrium payoff) is contained in the set for which a seller observing $s'_S \neq s_S$ strictly benefits, then $B$, upon observing $p$, assigns probability zero to $s_S$. An informal discussion of the role played by the refinement in our analysis is postponed to section 5.

Before characterizing the equilibrium, it is worth mentioning the first best benchmark. With full information trade only occurs if the seller is of type $H$ and the total
expected surplus is \( W^{FI} \equiv \lambda [u_H - v_H] \). Since \( S \) is able to extract all the surplus from \( B \) under full information, \( S \)'s profits are equal to \( W^{FI} \).

### 3.1 The limiting case: \( S \) is perfectly informed and \( B \) has no private information

This section illustrates how the market breaks down in the limiting case in which the seller has perfect information and the buyer only observes the price. In the next section we consider the general case in which both \( B \) and \( S \) receive a noisy signal.

Let \( \mu(q|p) \) denote \( B \)'s beliefs on the quality of the good upon observing \( p \).\(^7\) The application of NWBR is straightforward in this case. Consider a deviation \( p \). If the set of \( B \)'s mixed best responses for which type \( H \) strictly benefits (relative to her equilibrium payoff) contains the set for which type \( L \) weakly benefits, then \( \mu^*(L|p) = 0 \).

**Proposition 1.** There is a unique NWBR-refined equilibrium outcome, and it is such that no trade occurs.

The proof uses two arguments. First, there is no separating equilibrium in which trade occurs. Intuitively, type \( L \) can only trade by mimicking type \( H \) so that separation is never incentive compatible. Second, no pooling or hybrid equilibrium survives NWBR. If in a candidate equilibrium a price \( p \) is announced by both types, a type \( H \) seller would always prefer to deviate to a price higher than \( p \) in order to signal her type. The intuition here is that trading at \( p \) is relatively more costly for type \( H \) since \( v_H > v_L \). Therefore, the potential gain from a deviation \( p' > p \), relative to the equilibrium payoff, is higher for type \( H \) than for type \( L \). It follows that type \( H \) is “more seemly” to benefit from the deviation. Hence, \( B \) should believe that the deviation comes from type \( H \). This in turn implies that it is always profitable to deviate to some \( p' \), thus destroying any pooling or hybrid equilibria.

Differently from Akerlof (1970), the market breakdown is not induced by a downward pressure on the price due to adverse selection. Here, the market breakdown is caused by an upward pressure on the price driven by signaling concerns. The main effects at work can be metaphorically described as an “upward race”. Given pooling,

\(^7\)We use the simplified notation \( \mu(q|p) \) since \( S \) is perfectly informed and \( B \) observes only the price.
type $H$ increases her price to differentiate himself from type $L$. Given separation, type $L$ increases her price to mimic type $H$. When the price reaches $u_H$, no further upward deviation is possible, since $B$ would never buy at higher prices. However, if both types announce $p = u_H$, $B$ replies by not buying. Hence, the only refined equilibrium is one in which the market breaks down. Clearly, this equilibrium is sustained by off-equilibrium beliefs which give a weight large enough to type $L$. This is robust to NWBR since both types make zero surplus. It follows that both types have the same incentive to deviate to any $p > v_H$.

It is immediate to show that the NWBR-refined outcome survives (unobserved) small perturbations of the buyer’s valuation, as in Ellingsen (1997). This suggests that the market breakdown result is not a consequence of the inelasticity of demand.

### 3.2 The general case: both $S$ and $B$ are privately informed

In this section we consider the general case in which both $S$ and $B$ observe a private signal about the quality of the good. The only difference with respect to the limiting case is that we allow both parties to have some private information. Every other aspect of the model, including the structure of the game is unchanged. The signals $s_B \in [s_B, \bar{s}_B]$ and $s_S \in [s_S, \bar{s}_S]$ received by $B$ and $S$ are assumed to be independently drawn conditional on quality $q$. The density and the cumulative functions for $s_B$ are denoted with $f(s_B|q)$ and $F(s_B|q)$, respectively. Similarly, the density and the cumulative functions for $s_S$ are denoted with $g(s_S|q)$ and $G(s_S|q)$, respectively. We impose the following restrictions on $f(\cdot|q)$ and $g(\cdot|q)$:

**Assumption 1.** $f(\cdot|q)$ and $g(\cdot|q)$:

1. Are continuous with full support;

2. Satisfy the Monotone Likelihood Ratio Property (MLRP):

$$\frac{f(s_B|H)}{f(s_B|L)} \text{ is increasing in } s_B$$

$$\frac{g(s_S|H)}{g(s_S|L)} \text{ is increasing in } s_S$$

and the above ratios have full support $(0, \infty)$.
The assumption on the likelihood ratios is standard\(^8\) and greatly simplifies the task. On the other hand, it also implies that extreme signal realizations tend to leave agents with little uncertainty on the true state of nature.

The main result of this section is the following.

**Proposition 2.** Assume that \(f(\cdot|q)\) is non-degenerate and satisfies assumption 1. Then in both of the two following cases:

1. \(g(\cdot|q)\) is non-degenerate and satisfies assumption 1,
2. \(S\) is perfectly informed so that \(g(\cdot|q)\) is degenerate,

there is a unique NWBR-refined equilibrium outcome, and this is such that no trade occurs.

In order to understand the intuition behind the result, it is worth noticing that it applies independently of the quality of the information available to the parties. Not only does proposition 2 apply to the case in which \(S\) receives a more precise signal than \(B\), it also applies to the case in which \(B\) observes a more precise signal than \(S\). In particular, proposition 2 says that for all non-degenerate distributions of \(B\)'s signal, the only robust equilibrium is such that no trade ever occurs, even when \(B\) has very precise information. In other words, trade collapses independently of whether the seller is better informed than the buyer. It is sufficient that the seller have some private information which could be potentially disclosed through the price choice. It follows that the impossibility of trade is here a more pervasive feature than in a standard adverse selection model in which agents are price-takers. The ability of the seller to convey information through prices amplifies the effect of adverse selection rather than reducing it.

As in the previous section, the result relies on the impossibility of separating equilibria and on the fact that pooling and hybrid do not pass NWBR. We sketch the argument for the case in which \(g(\cdot|q)\) is non-degenerate. In this case, \(S\)'s valuation is a function of her signal \(s_S\). It can be shown that if \(S\) weakly prefers to announce \(p\) to \(p' < p\) when she has a low valuation for the good, then she strictly prefers to announce \(p\) when she has a high valuation. Intuitively, this sorting condition ensures that higher

\(^8\)See for instance Benabou and Tirole 2003.
prices correspond to higher valuations by $S$. $B$ decides to buy only if the realization of his signal $s_B$ is above a certain threshold which depends on the price announced by $S$.

Consider first a candidate separating equilibrium, i.e. a situation in which $S$’s strategy is a one to one mapping from realizations of her signal into prices. Lower prices have two opposite effects on $B$’s willingness to buy. They reduce $B$’s threshold since he has to pay less for the good (direct effect), and increase $B$’s threshold since they signal that $S$ has a low valuation (signaling effect). For a separating equilibrium to be viable, the direct effect should always prevail. In other words, the probability to sell must be decreasing in the price. Otherwise, $S$ could profit from mimicking (i.e. from setting a relatively high price) when her valuation is low. However, since $v_L < u_L$, for low enough realizations of $S$’s signal, the separating price should drop below $v_L$ to ensure that $B$ buys. This can never happen in equilibrium, since $S$ makes a loss from any $p < v_L$. Hence, for bad enough realizations of the signal, the signaling effect prevails. No separating equilibrium is therefore possible.

Let us now focus on pooling and hybrid. In these equilibria, the same price $p$ is announced for different valuations of $S$. The sorting condition implies that a seller with a given valuation prefers to deviate to a higher price for any $B$’s reply such that a seller with a lower valuation weakly prefers to deviate. Intuitively, upon observing a deviation to a higher price, $B$ infers that the deviation comes from the type of seller who gains more from deviating, relative to her equilibrium payoff (NWBR). This is always the type with the highest signal among the signals for which $S$ announces $p$. Clearly, these out of equilibrium beliefs provide $S$ with a strong incentive to deviate to a higher price.

These results point out an intrinsic instability of the equilibrium: as soon as the information is perturbed, the equilibrium outcome radically changes. To see this, consider the case in which the precision of the private signals is almost perfect so that the game approximates the full information game. Since proposition 2 holds for all non-degenerate distributions, the amount of trade is discontinuous with respect
to the precision of the signals. Whenever a negligible amount of noise is introduced, the amount of trade drops from the full information level to zero. In particular, since proposition 2 holds even if $S$ has perfect information, it is sufficient a small perturbation of $B$'s information for trade to collapse entirely.

4 The Price-Setting Buyer Model

The results discussed in the previous section apply to the case in which $B$ sets the price under symmetric restrictions on gains from trade. We maintain the assumption that $v_H > v_L$ and $u_H > u_L$, but we now assume that potential gains from trading the low quality are positive and potential gains from trading the high quality are negative, i.e. $u_L > v_L$ and $u_H < v_H$. There are several instances in which trading extremely high qualities may be inefficient. For example, consider an employer who wants to hire a worker for a job that does not require any particular skill. It is reasonable to assume that for unskilled workers the (opportunity) cost of supplying labor is exceeded by the benefits that firms derive from hiring them. The same might not be true for high ability workers. Another application is the “task attractiveness” model discussed in section 5.

We show that this game is symmetric to the one analyzed in the previous sections where the price setting party was the seller. Accordingly, all the results from the previous section apply.

The timing of the game is as follows. At stage 0, Nature draws $q \in \{H, L\}$. At stage 1, either $B$ or both $S$ and $B$ exogenously receive some private information about the quality of the good. At stage 2, $B$ announces a price $p$ at which he is willing to trade. At stage 3, $S$ observes $p$ and chooses whether to sell or not. Finally, at stage 4, payoffs are realized.

Proposition 3. When $B$ sets the price the results are symmetric to those of propositions 1 and 2. In particular, if $g(\cdot | q)$ is non-degenerate and satisfies assumption 1, in both of the following cases:

1. $f(\cdot | q)$ is non-degenerate and satisfies assumption 1,
2. $B$ is perfectly informed so that $f(\cdot | q)$ is degenerate,
there is a unique NWBR-refined equilibrium outcome and is such that no trade occurs.

The proof relies on a symmetry argument. The game in which $B$ sets the price and $u_H < v_H$ is symmetric to the game in which $S$ sets the price and $u_L < v_L$.

The intuition is again that separating equilibria are not possible. In a separating equilibrium, $B$ should announce a higher price whenever he has a higher valuation. As before, lower prices have both a negative (direct) effect and a positive (signaling) effect on $S$’s willingness to trade. In order to give incentive to $B$ to reveal his valuation through the price, the direct effect should always prevail. However, so long as $u_H < v_H$, this cannot happen when $B$ observes sufficiently high realizations of his signal and thus has a sufficiently high valuation. The separating price that $B$ should set to make $S$ willing to sell would be above his utility from a high quality good, $u_H$. Clearly enough, $B$ would never find it optimal to set such a price. This makes separation impossible to achieve.

As for pooling and hybrid equilibria, they never pass NWBR. Given pooling at some price, the lower the signal received by $B$, the higher is $B$’s incentive to deviate to a lower price. NWBR forces $S$’s off-equilibrium beliefs to reflect these differences in incentives. This in turn makes the deviation profitable for $B$.

Intuitively, $B$ wants to lower the price when his valuation is low in order to differentiate himself. On the other hand, he wants to lower the price when his valuation is high to mimic a low valuation buyer. This ensures that the price eventually reaches $v_L$ at which no trade occurs.

5 The Role of Price Rigidities

The previous sections show that when the party setting the price has private information, market breakdown may become a pervasive phenomenon. If one believes that NWBR is a sensible refinement, then our analysis suggests that absence of trade is the most reasonable outcome. Even if one considers NWBR an excessively strong refinement that may sometimes eliminate reasonable equilibria, our result still shows that the equilibrium with no trade cannot be dismissed as the product of ad-hoc
off-equilibrium beliefs.

Seen from a different angle, however, the result is not entirely satisfactory. Casual observation suggests that there are many instances in which trade is observed under conditions that reasonably resemble those assumed here. From a theoretical viewpoint, it is easy to check that there exist pooling equilibria in which trade occurs.\footnote{These always exist in the general case in which both the price setting and the price taking parties have private information. In the limiting case in which only the price setting party is privately informed they may exist if $\lambda$ is high enough.} These equilibria are however characterized by relatively pessimistic out of equilibrium beliefs whenever the price taker observes a price different from the equilibrium price. For instance, if the seller deviates to a price higher than the pooling price, beliefs that support the equilibrium should be such that the quality expected by the buyer is low enough. By contrast, beliefs robust to NWBR have the intuitively appealing feature that higher prices induce expectations of higher quality.

What approach should one trust? Should one accept the strong prediction of market breakdown or should one instead acquiesce to the indeterminacy generated by arbitrary out of equilibrium beliefs? Ideally, one would like to have a theory determining the conditions that may favor trade, capable of providing empirical predictions. In this section we introduce an extension of the model that addresses this issue. We focus on an extreme feature of the model, namely the assumption that prices are perfectly flexible and can be set and changed at no cost by the price setting party. The result of complete market breakdown turns out to be not robust to the introduction of price rigidities. In particular, we show that:

1. whenever prices are not perfectly flexible, there is a NWBR-refined equilibrium in which the expected amount of trade is positive,

2. in this equilibrium, prices are completely uninformative,

3. the expected amount of trade, expected profits, and expected surplus from trade are weakly increasing in the degree of price rigidity.

Intuitively, market breakdown essentially stems from lack of commitment devices.
The price setting party would be able to trade by committing to prices that do not reveal her information.

Before returning to the model, we discuss how the analysis can be applied to the market for professional services and to compensation policies in the presence of “hidden costs” of monetary incentives discussed by Benabou and Tirole (2003). Here the focus is on institutional price rigidities, rather than on rigidities induced by technological constraints.

**Professional Bodies**

There is a number of professional services for which the value of a low quality service is higher for the seller than for the buyer. Practitioners bear a positive cost to provide a low quality service whereas the utility for the customer may be zero or even negative, as in the case of a physician making a wrong diagnosis or a dentist using non-sterile instruments. In many countries, however, practitioners do not set fees directly. A professional association or the government often set fees in their stead. In many cases, they are even forbidden to advertise their fees. Independently of whether fee scales are considered as mandatory or just “recommended” by the professional association, the members tend to consider them as binding.\(^\text{10}\) Since the professional body typically sets the fees for a large number of members, these convey no information about the quality of an individual practitioner.

Economists have traditionally viewed professional bodies as associations enforcing cartel prices in order to prevent competition among members (Matthews 1991). We do not deny that this is a sensible explanation for the existence of professional bodies. Yet, our results provide a complementary explanation which can shed light on some features of professional bodies that are not fully explained by the traditional view.

For instance, our results may explain why this price setting function of professional

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\(^{10}\)Shinmick and Stephen (2000), citing the Monopolies and Mergers Commission, report “... although disciplinary action could not be taken specifically for breach of a recommended scale, the fact that the fees charged were not in accordance with the scale might in some circumstances be quoted in support of a charge of breach of some other rule .... such that the established practitioner would not depart more readily from a ‘recommended’ scale than from a mandatory scale.”
associations is more crucial in service markets such as health than in other markets. Moreover, they may explain why professional associations sometimes set maximum fees.\footnote{For instance, the Professional Board for Psychology in South Africa sets “Ethical Tariffs” that provide a ceiling to the rates chargeable by the members.} These are clearly of no help in preventing undercutting. By contrast, in our model, a maximum price between buyer and seller valuation for the high quality could produce a positive amount of trade.

Kranton (2003) shows how the presence of professional associations, by restricting competition, may provide incentives to maintain reputation with consumers by producing high quality. In her case, the role of professional associations is that of reducing competition by restricting entry or output. Our emphasis is mainly on the price setting function carried out by professional associations. As we argue, this might be valuable even in the absence of competitive pressures.

**Task Attractiveness (Benabou and Tirole 2003)**

The price setting buyer model has an interpretation that shares similarities with Benabou and Tirole (2003) model of intrinsic and extrinsic incentives. In their model, a principal has private information about the cost of undertaking a particular task. They show that the principal may not want to give high powered monetary incentives to the agent. Excessively strong incentives may signal to the agent that the task is particularly difficult, risky, or unpleasant. As a result, monetary incentives may backfire.

Consider now the price setting buyer model discussed in section 4. Suppose that an employer (buyer) wants to hire a worker (seller) for a particular task. There are two types of tasks: high cost tasks (type $H$) and low cost tasks (type $L$). The employer derives utility $u_q$ (with $u_H > u_L$) from completion of a type $q$ task. The cost to the worker of undertaking a type $q$ task is $v_q$ (with $v_H > v_L$). The employer has private information on whether the task is low or high cost. We assume that she is constrained to offering a simple contract that involves a transfer (i.e. $p$) conditional on completion of the task. Consistent with our story, suppose that, from a social
viewpoint, only the low cost task should be performed, i.e. $u_H < v_H$ and $u_L > v_L$. As shown in section 4, the unique NWBR-refined equilibrium involves no trade between the employer and the worker. This can be interpreted as an extreme case of "hidden costs" of monetary incentives (negative reinforcement effect). For example, suppose that someone you suspect might be involved in drug trafficking asks you to deliver a sealed pack to a third party. Any attempt to lure you into accepting the task by offering disproportionate amounts of money will only increase your suspicions about the content of the pack. As a result, trade may collapse.

More generally, our results suggest that the employer could be better off if he could commit to a uniform wage policy, i.e. offer the same transfer independently of the information he has about the task. Indirect evidence supporting this idea comes from the compensation of subjects taking part in medical experiments. Dickert et al. (2002) survey compensation guidelines of organizations involved in biomedical experiments. They find that more than 80% of the organizations paid subjects for their time and inconvenience (which are arguably observable by the subject). By contrast, only 32% paid the subjects to compensate for the risk associated with taking part in the experiment (on which the researcher typically has private information).

The next section formally analyzes the effect of price rigidities.

5.1 The Price-Setting Seller Model with Price Rigidities

We illustrate the result for the case of a price setting seller. The result clearly holds in the symmetric case when the buyer sets the price. For simplicity, we assume that the seller perfectly observes her type. Note that this assumption works against the result we are set to prove since the potential for information revelation through prices is maximum when the price setting party is perfectly informed. The simplest way to allow for price rigidities in this setup is to assume that $S$ is partially able to commit to a price, say $p_0$, before receiving information about her type. After knowing her type, $S$ may decide to stick to the initial price or to switch to a different price. If trade
occurs at a price different from $p_0$, then $S$ incurs a cost $c \geq 0$. Thus, $c$ is a measure of the rigidity of the price and, as we shall see, of $S$’s ability to commit to prices that do not reveal information. The timing of the game can be summarized as follows:

- $t = 0$, $S$ chooses a price $p_0 \in [0, u_H]$ that is observed by $B$.
- $t = 1$, Nature draws a type $q \in \{H, L\}$ for $S$ that is observed by $S$.
- $t = 2$, $S$ announces a price $p_2 \in [0, u_H]$ (we say that $S$ “switches” if $p_2 \neq p_0$ and that $S$ “sticks” if $p_2 = p_0$).
- $t = 3$, $B$ observes the price $p_2$, observes a signal $s$, and chooses whether to buy or not. If trade occurs at a price different from $p_0$, $S$ incurs a switching cost $c$.
- $t = 4$, payoffs are realized.

We also assume that, at time $t = 2$, $S$ can always choose not to put the good on offer.

Three implicit assumptions in this setup deserve further discussion:

1. There exists an initial-price setting stage in which $S$ is uninformed.
2. The cost of switching to a different price is conditional on trade taking place.
3. $B$ is able to observe the initial price $p_0$.

The first assumption can be seen as a shortcut to describe two different situations:

- **The type, $q$, is interaction-specific:** Over time, $S$ may be involved in a number of similar private transactions, each with a different buyer. Then the setup just described would roughly correspond to a situation in which $S$’s type varies across interactions and $S$ can (partially) commit to having a unique price for all transactions. In the case in which the buyer sets the price this may correspond to an employer committing to offering the same wage for all tasks.

- **The type, $q$, is agent-specific:** In some cases, $S$’s type is highly correlated across different interactions, as in the example of a practitioner’s ability. Hence,
there is no proper stage in which $S$ is uninformed about her type. In this case, our setup would correspond to a situation in which an association of producers sets a unique price for all members at $t = 0$. This is for instance the case of a professional body setting fees for a large number of practitioners. In this case, fees cannot convey information on the ability of a single practitioner. The price rigidity here would stem from the threat of disciplinary action whenever a member charges a different fee.

Consider now the second assumption. Although some sources of rigidity involve fixed costs, many costs, such as the cost of changing price tags on items, vary according to the volume of trade.\footnote{Zbaracki et al. (2000) report that, for an industrial firm, more than 73% of the total cost of changing prices consisted in “customer costs” which are in large part variable costs.} A natural way to incorporate these costs in our simplified model is to assume that the cost of switching to a different price is proportional to the probability of trade. Hence, the assumption of a trade-contingent switching cost can be interpreted as a surrogate for variable costs. This broad interpretation of the model is however not necessary. Specific sources of price rigidity are compatible with a stricter interpretation. These include:

- administrative/accounting costs (e.g. cost of keeping records of transactions at different prices),
- litigation costs or social disapproval attached to behavior perceived as discriminatory or unfair.\footnote{King and Narayandas (2000) report how a Coca Cola’s project to install vending machines that could charge different prices according to the temperature met negative consumers’ reactions and was quickly dropped.} This is for instance the case of a firm paying different compensations for tasks that are apparently similar.

The example in which a professional body may take disciplinary action against a member charging a different fee also falls into this category. It is extremely difficult to make the case for a disciplinary action if no transaction has occurred.

Finally, the assumption that $B$ observes the initial price is realistic in many settings and is not necessary for the main result. Even if $p_0$ were not observed, $B$ would be
able to perfectly predict it in equilibrium.

A NWBR-refined equilibrium of the entire game requires that: a) given an initial price $p_0$, the signaling game starting at $t = 1$ is in a NWBR-refined Perfect Bayesian Equilibrium, b) $p_0$ is chosen at $t = 0$ as to maximize $S$'s expected profits given the equilibrium of the signaling game starting at $t = 1$. Intuitively, the signaling game starting at $t = 1$ determines the outcome for a given initial price $p_0$. At $t = 0$, $S$ chooses $p_0$ as to maximize her expected profits given the outcome of the signaling game. The next proposition shows that a NWBR-refined outcome contemplates a positive amount of trade only if the equilibrium of the signaling game is such that no switching occurs at $t = 2$. Hence, $S$’s problem at $t = 0$ is to choose a level for $p_0$ that prevents ex-post switching.

**Proposition 4.** There always exists a NWBR-refined equilibrium outcome of the entire game such that no trade occurs. This is the unique equilibrium outcome when $c = 0$. When $c > 0$, there also exists a NWBR-refined equilibrium of the entire game in which trade occurs with positive probability. This is a pooling equilibrium in which $S$ never switches (i.e. $p_2 = p_0$). This equilibrium outcome involves positive expected profits for $S$ and positive expected total surplus from trade. The probability of trade, $S$’s expected profits, and expected total surplus are weakly increasing in $c$.

One might imagine that the result of positive trade is driven by the fact that, in the presence of price rigidities, $S$ is willing to deviate to higher prices only when she is able to recoup the switching cost through the price increase. Hence, if the initial price $p_0$ is close enough to $u_H$, $S$ does not bother to deviate. This intuition is however misleading. Even if the price increase is not enough to repay the switching cost, $S$ would still want to deviate to a higher price if this could increase her chances to trade. The effect of $c$ here is more subtle and works through the information channel. When $c$ is positive, deviations to prices slightly above $p_0$ are not interpreted by $B$ as a signal of high quality. The reason is that both types could benefit from the deviation only if the probability of trade increased as a consequence. However, if the probability of trade increases, type $L$ benefits more than type $H$. As a result, for the deviation to be a credible (out of equilibrium) signal of high quality, the price increase must be relatively large. But then, at $t = 0$, $S$ can always choose a price $p_0$ high enough to
ensure that no deviation can be interpreted as signal of high quality. Clearly enough, the smaller is \( c \), the closer \( p_0 \) must be to \( u_H \) to discourage deviations. As a result, when \( c \) is very small (flexibility is high), the expected amount of trade is also small.

Given that trade may occur in equilibrium, \( S \) makes positive expected profits since the price at which trade occurs is always greater than or equal to \( v_H \). Since the equilibrium is characterized by pooling, \( B \) uses a threshold strategy on the realization of her private signal to decide whether to buy or not. For the threshold to be optimal, \( B \) should not make expected losses in equilibrium. As a result, expected surplus generated by trade is always positive.

The comparative static result on \( c \) can be easily understood once one considers that \( c \) is a measure of \( S \)'s ability to commit. Take for instance an increase of \( c \). This widens the set of prices that are not susceptible to be switched ex-post. \( S \) is thus able ex-ante to pick \( p_0 \) from a larger set which includes prices that were not available before because they were too low to prevent switching. The increased choice always (weakly) increases expected profits. Moreover, since the ex-ante optimal price either decreases or stays constant, while the pool of qualities is unchanged, the amount of trade and total surplus must (weakly) increase as a result.

5.2 Discussion and Extensions

The comparative static result suggests that price rigidities have unambiguous non-negative effects on trade, welfare, and profits. On the other hand, our simple model focuses on a good that has only one dimension of quality which is imperfectly observed by one party. Real world goods or services typically have multiple characteristics some of which might become manifest to both parties at the trading stage. In these contexts, the effect of price rigidities may be more ambiguous. The contribution of this paper is to highlight a benefit of price rigidities and to suggest that the optimal level of rigidity is unlikely to be zero when some characteristic of the good is imperfectly observed. This has interesting implications. A mix of technological progress and deregulation has recently contributed to a great increase in price flexibility in particular markets.
(e.g. air travel, car rental, internet retailing\textsuperscript{14}). The extent to which these innovative pricing systems will become commonplace in the future is debated. Our result suggests that some degree of price rigidity is likely to survive in markets where information is imperfect, interactions are infrequent, and the exchange of low quality goods may destroy value.

We now briefly return to the analysis of professional bodies. This can be extended to account for characteristics of the good that are perfectly observable by all parties (in addition to the privately observed quality). In the presence of observable characteristics, the price setting party would find it optimal to have prices that reflect these characteristics. On the other hand, she would like to commit to prices that do not reveal her private information. As we have seen, a possible solution for the price setting party would be to delegate pricing decisions to an association such as a professional body, which would set a price for all members. The price would thus reflect observable characteristics of members as much as possible, but not their private information. The value to members of such an association would be higher the more homogeneous are the members in terms of observable characteristics. As a result, the professional body might find it optimal to screen its members according to their observable characteristics, for instance by imposing entry requirements. By contrast, access to information about the privately known characteristics of its members could actually damage the professional body since it might reduce the value of its price setting function.

\section{Conclusions}

We considered a model of monopoly/monopsony with two-sided private information. Our results showed that, under empirically relevant assumptions, the only NWBR-refined equilibrium outcome involves complete market breakdown. We also found that a positive amount of trade is restored in the presence of a positive degree of price

\textsuperscript{14}For example, Brynjolfsson and Smith 2000 report how internet retailers are willing to make smaller price changes than conventional retailers.
rigidity. Our results can be applied to several areas. The model can be adapted to capture empirically relevant features of professional bodies. It can also be used to analyze the incentives to invest in technologies that increase price flexibility. Finally, it predicts that an employer may find it optimal to commit to a uniform wage policy in the presence of “hidden costs” of monetary incentives.
A Appendix

A.1 Proof of Proposition 1

The proposition is proved in three steps. We first prove that there is no separating equilibrium in which trade occurs. Then we prove that no NWBR-refined equilibrium in which trade occurs can involve any type of pooling. Finally, we prove that the outcome involving market breakdown is consistent with a NWBR-refined equilibrium.

**Lemma 1.** There is no fully separating equilibrium in which trade occurs.

*Proof.* Let $P_L$ and $P_H$ be the set of prices announced in equilibrium by type $L$ and $H$ respectively. If $P_L \cap P_H = \emptyset$, type $L$ is never able to trade. However, type $L$ would benefit from trading at any $p \in P_H$ given that $p$ is optimal for type $H$, since $v_L < v_H$. Hence, type $L$ would always try to mimic type $H$. □

**Lemma 2.** No NWBR-refined equilibrium in which trade occurs can be a pooling or a hybrid equilibrium.

*Proof.* Consider a pooling/hybrid equilibrium with trade. Let $\hat{p}$ be some price at which pooling occurs. Suppose that type $H$ announces $\hat{p}$ with probability $\beta_H \in (0, 1]$ and type $L$ announces $\hat{p}$ with probability $\beta_L \in (0, 1]$. Then $B$’s expected utility at $\hat{p}$ is:

$$
\frac{\lambda \beta_H}{\lambda \beta_H + (1 - \lambda)\beta_L} - \frac{(1 - \lambda)\beta_L}{\lambda \beta_H + (1 - \lambda)\beta_L} - \hat{p} \tag{A.1}
$$

Let $\hat{\alpha}$ be the (possibly 1) probability with which $B$ buys at $\hat{p}$. Payoff at $\hat{p}$ for $H$ is:

$$
\hat{\alpha}(\hat{p} - v_H) \tag{A.2}
$$

payoff for $L$ is:

$$
\hat{\alpha}(\hat{p} - v_L) \tag{A.3}
$$

Consider a deviation $p$ and assume that the buyer, upon observing $p$, buys with probability $\alpha$. Then type $L$ is eliminated according to NWBR if, for all $\alpha$ such that

$$
\alpha(p - v_L) \geq \hat{\alpha}(\hat{p} - v_L), \tag{A.4}
$$

the following holds:

$$
\alpha(p - v_H) > \hat{\alpha}(\hat{p} - v_H) \tag{A.5}
$$

Consider an upward deviation $p > \hat{p}$ and suppose that (A.4) holds. It is easy to verify that the following is true:

$$
\frac{\alpha}{\hat{\alpha}} \geq \frac{\hat{p} - v_L}{p - v_L} > \frac{\hat{p} - v_H}{p - v_H} \tag{A.6}
$$

The first inequality comes from (A.4) while the second comes from $p > \hat{p}$. Note that the above expression implies that (A.5) is satisfied whenever (A.4) is satisfied. Therefore, the low quality can be always eliminated from the deviation. As a result, the high quality has always incentive to deviate unless $\hat{p} = u_H$. On the other hand, trade cannot occur at $\hat{p} = u_H$ so long as $\beta_L > 0$. Hence there cannot be a pooling/hybrid equilibrium in which trade occurs. □
Lemma 3. There is a NWBR-refined equilibrium in which no trade occurs.

Proof. Consider a situation in which $S$ always announces $p = u_H$ and $B$ never buys at any price. This is clearly an equilibrium if $B$ believes that deviations $p \geq v_H$ emanate from type $L$. It is also robust to NWBR since, for a deviation $p \geq v_H$, the set of $B$'s best responses that make type $L$ willing to deviate coincides with the set of best responses that make type $H$ willing to deviate. Therefore, type $L$ cannot be eliminated.

A.2 Proof of Proposition 2

We start by giving the proof for the case in which $S$ is perfectly informed and $B$ observes a private signal (case 2 in proposition 2). Then we turn to the case in which both $B$ and $S$ observe private signals (case 1 in proposition 2).

A.2.1 $S$ is perfectly informed and $B$ observes a private signal

We start off by showing that no separating equilibrium in which trade occurs exists. Then we show that no pooling or hybrid equilibrium in which trade occurs passes NWBR. Finally, we prove that there is a NWBR-refined equilibrium in which trade does not occur.

Lemma 4. There is no separating equilibrium in which trade occurs.

Proof. In a separating equilibrium, $B$ always discards her private signal as equilibrium prices are fully informative. The proof of lemma 1 hence applies and no separation is possible.

Lemma 5. No pooling-hybrid equilibrium in which trade occurs survives NWBR.

Proof. Assume that trade occurs in a pooling or hybrid equilibrium. Suppose as before that pooling occurs at $\hat{p} < u_H$. A high quality seller announces $\hat{p}$ with probability $\beta_H \in (0,1]$ and a low quality seller announces $\hat{p}$ with probability $\beta_L \in (0,1]$. $B$ observes price $\hat{p}$ and receives a signal $s$ (we omit the $B$ subscript from the signal). $B$’s expected utility from buying at $\hat{p}$ is:

$$\frac{\lambda \beta_H f(s|H)}{\lambda \beta_H f(s|H) + (1 - \lambda) \beta_L f(s|L)} u_H + \frac{(1 - \lambda) \beta_L f(s|L)}{\lambda \beta_H f(s|H) + (1 - \lambda) \beta_L f(s|L)} u_L - \hat{p} \quad (A.7)$$

Expected utility is nonnegative if:

$$\frac{f(s|H)}{f(s|L)} \geq \frac{(1 - \lambda) \beta_L \hat{p} - u_L}{\lambda \beta_H u_H - \hat{p}} \quad (A.8)$$

Notice that the LHS is an increasing function of $s$ and the RHS is positive. Given the full support assumption, there always exists a threshold $s^* \in [\underline{s}, \overline{s}]$ such that $(A.8)$ holds if $s \geq s^*$ and does not hold if $s < s^*$. Hence, $B$’s threshold strategy is to buy if $s \geq s^*$ and not to buy for $s < s^*$. $S$’s payoff is:

$$[1 - F(s^*|q)](\hat{p} - v_a) \quad (A.9)$$
where \( q \in \{H, L\} \). Suppose now that \( B \) observes a deviation \( p > \hat{p} \). Upon observing \( p \), \( B \) uses a a threshold \( s^D \) (see Benabou and Tirole 2003 on this way to use NWBR). According to NWBR, beliefs assign probability zero to type \( L \) (i.e. type \( L \) can be eliminated) if the set of values for \( s^D \) that make her weakly benefit from the deviation is contained in the set of values that make type \( H \) strictly benefit. Type \( L \) would (weakly) benefit whenever:

\[
[1 - F(s^D | L)](p - v_L) \geq [1 - F(s^L | L)](\hat{p} - v_L) \tag{A.10}
\]

Type \( L \) can be eliminated if, whenever (A.10) holds, the following also holds:

\[
[1 - F(s^D | H)](p - v_H) > [1 - F(s^L | H)](\hat{p} - v_H) \tag{A.11}
\]

Notice that (A.11) is always verified whenever \( s^D \leq s^* \) since type \( H \) would profit from a higher price and a lower threshold (which implies a higher probability to sell). Consider then \( s^D > s^* \). It is immediate to verify that, for a deviation \( p > \hat{p} \), (A.11) is always satisfied when (A.10) holds so long as:

\[
\frac{1 - F(s^D | H)}{1 - F(s^* | H)} \geq \frac{1 - F(s^D | L)}{1 - F(s^* | L)} \tag{A.12}
\]

Rewrite the above as:

\[
(1 - F(s^D | H))(1 - F(s^L | L)) - (1 - F(s^* | H))(1 - F(s^D | L)) \geq 0 \tag{A.13}
\]

The derivative of the above expression with respect to \( s^D \) is

\[
-f(s^D | H)(1 - F(s^* | L)) + f(s^D | L)(1 - F(s^* | H)) \tag{A.14}
\]

so that the LHS of equation (A.13) is increasing whenever:

\[
\frac{f(s^D | H)}{f(s^D | L)} < \frac{1 - F(s^* | L)}{1 - F(s^* | H)} \tag{A.15}
\]

and is decreasing whenever the reverse inequality holds. Given the MLRP (which implies that \( \frac{f(s^D | H)}{f(s^D | L)} \) is an increasing function), the LHS of inequality (A.13) must be an increasing-decreasing function (i.e. increasing for small values of \( s^D \) and decreasing beyond a certain value).

We note that the limits of (A.13) for \( s^D \to s^* \) and \( s^D \to \bar{s} \) are both zero. Since the LHS of inequality (A.13) is an increasing-decreasing function which converges to zero as \( s^D \) moves toward the bounds of \((s^*, \bar{s})\), it follows that it cannot be negative in \((s^*, \bar{s})\). Hence, (A.12) holds and the low type can be eliminated for any deviation \( p > \hat{p} \). Since type \( L \) can be eliminated, for deviations \( p < u_H \), \( B \) would always buy with probability one. But then, it is always optimal for \( S \) to deviate to prices \( p \in (\hat{p}, u_H) \), which implies that there cannot be any pooling or hybrid equilibrium with trade. \( \square \)

**Lemma 6.** There always exists a NWBR-refined equilibrium in which trade does not occur.

**Proof.** The proof is identical to that of lemma 3. Both type \( H \) and type \( L \) announce \( u_H \) and \( B \) selects a threshold equal to \( \bar{s} \). \( \square \)
A.2.2 Both S and B receive a private signal

We start by proving that B follows a threshold strategy. Then we analyze the sorting condition. The proof could be complicated by the fact that the sorting condition holds only for prices above \( v_H \), while, in principle, trade could also occur at lower prices. Therefore, our strategy consists in showing that any equilibrium with trade must involve a degree of pooling at some price \( p \geq v_H \). Then, we show that this would violate NWBR.

**Lemma 7.** Fix a price \( p \in (v_L, u_H) \). Then, B follows a threshold strategy \( s_B^* \in [\underline{s}_B, \overline{s}_B] \). The threshold \( s_B^* \) is equal to \( \underline{s}_B \) (resp. \( \overline{s}_B \)) if and only if, upon observing \( p \), B believes that S received realization \( \overline{s}_S \) (resp. \( \underline{s}_S \)).

**Proof.** Suppose that S announces price \( p \) for some set of signal realizations. Since \( s_S \) is, conditionally on \( q \), independent of \( s_B \), the price \( p \) is also independent of \( s_B \) conditionally on \( q \). Hence, it is easy to show that:

\[
\Pr(q|s_B, p) = \frac{f(s_B|q) \Pr(q)}{\sum_{q \in \{H,L\}} f(s_B|q) \Pr(q)} \quad (A.16)
\]

where \( \Pr(q) \) is equal to \( \lambda \) (resp. \( 1 - \lambda \)) when quality is \( H \) (resp. \( L \)) and \( P(\cdot|q) : \mathbb{R}^+ \to [0,1] \) is a conditional probability function consistent with \( S \)'s strategy. B’s expected payoff from buying at \( p \) is therefore:

\[
\frac{\lambda f(s_B|H) P(p|H)}{\lambda f(s_B|H) P(p|H) + (1 - \lambda) f(s_B|L) P(p|L)} u_H + (1 - \lambda) f(s_B|L) P(p|L) u_L - p \quad (A.17)
\]

so that B’s payoff is nonnegative if:

\[
\frac{f(s_B|H)}{f(s_B|L)} \geq \frac{1 - \lambda}{\lambda} \frac{p - u_L}{u_H - p} \quad (A.18)
\]

From the MLRP, the LHS is an increasing function of \( s_B \). Notice that: 1) if \( p \) is announced for any realization of \( S \)'s signal \( s_B < \underline{s}_B \), then \( P(p|L) > 0 \), 2) if \( p \) is announced for any realization of \( S \)'s signal \( s_B > \overline{s}_B \), then \( P(p|H) > 0 \). It follows that, if \( p \) is announced for any \( s_B \in (\underline{s}_B, \overline{s}_B) \), the RHS of (A.18) is finite and strictly greater than zero. Given the full support assumption, in this case there always exists a threshold \( s_B^* \) such that B’s payoff is nonnegative for \( s_B \geq s_B^* \) and negative otherwise. Hence, B follows a threshold strategy which depends on \( p \).

We now turn to the limiting cases in which the RHS of (A.18) might go to zero or infinity. Suppose that there is a price \( p \in (v_L, u_H) \) that is only announced when the realization of \( S \)'s signal is \( s_S \). Then \( P(p|q) \) can be replaced with \( g(s_S|q) \). B’s threshold \( s_B^* \) must thus satisfy:

\[
\frac{f(s_B^*|H)}{f(s_B^*|L)} = \frac{1 - \lambda}{\lambda} \frac{p - u_L}{u_H - p} g(s_S|H) \quad (A.19)
\]

Suppose now that \( s_S = \underline{s}_S \). Since \( g(s_S|H)/g(s_S|L) \) is monotonically increasing and has full support \((0,\infty)\), the inverse ratio \( g(s_S|L)/g(s_S|H) \) must diverge to infinity when
Lemma 8. Given B’s threshold \( s^*_B \), S’s expected net payoff from announcing price \( p \) conditional on observing \( s_S \) is:

\[
(1 - F(s^*_B|H)) \Pr(H|s_S)(p - v_H) + (1 - F(s^*_B|L)) \Pr(L|s_S)(p - v_L) \tag{A.20}
\]

Proof. Notice that, since \( S \) is imperfectly informed, B’s decision to buy the good conveys information to \( S \) about the quality. Consider \( S \)’s payoff when, having received a signal \( s_S \), she chooses to put the good on offer at \( p \):

\[
[\Pr(H, s_B > s^*_B|s_S) + \Pr(L, s_B > s^*_B|s_S)] p + \Pr(H, s_B < s^*_B|s_S)v_H + \Pr(L, s_B < s^*_B|s_S)v_L
\]

The payoff if \( S \) chooses to keep the good is:

\[
\Pr(H|s_S)v_H + \Pr(L|s_S)v_L \tag{A.22}
\]

It is easy to show that \( \Pr(q, s_B > s^*_B|s_S) = (1 - F(s^*_B|q)) \Pr(q|s_S) \). Therefore, the expected net payoff is:

\[
[(1 - F(s^*_B|H)) \Pr(H|s_S) + (1 - F(s^*_B|L)) \Pr(L|s_S)] p + F(s^*_B|H) \Pr(H|s_S)v_H + F(s^*_B|L) \Pr(L|s_S)v_L - \Pr(H|s_S)v_H - \Pr(L|s_S)v_L \tag{A.23}
\]

Expression (A.20) follows. \( \square \)

Lemma 9. (Sorting) Let \( s_S \) and \( s'_S < s_S \) be two realizations of \( S \)’s signal. Let also \( p \) and \( p' < p \) be two prices, with \( p \geq v_H \). Then, whenever type \( s'_S \) weakly prefers \( p \) to \( p' \), type \( s_S \) strictly prefers \( p \) to \( p' \).

Proof. Type \( s'_S \) weakly prefers \( p \) when:

\[
(1 - F(s^*_B(p)|H)) \Pr(H|s'_S)(p - v_H) + (1 - F(s^*_B(p)|L)) \Pr(L|s'_S)(p - v_L) \geq 0
\]

\[
(1 - F(s^*_B(p'|H)) \Pr(H|s'_S)(p' - v_H) + (1 - F(s^*_B(p'|L)) \Pr(L|s'_S)(p' - v_L) \tag{A.24}
\]

where \( s^*_B(p) \) and \( s^*_B(p') \) are B’s thresholds when B observes prices \( p \) and \( p' \) respectively. Type \( s_S \) strictly prefers \( p \) when:

\[
(1 - F(s^*_B(p)|H)) \Pr(H|s_S)(p - v_H) + (1 - F(s^*_B(p)|L)) \Pr(L|s_S)(p - v_L) > 0
\]

\[
(1 - F(s^*_B(p'|H)) \Pr(H|s_S)(p' - v_H) + (1 - F(s^*_B(p'|L)) \Pr(L|s_S)(p' - v_L) \tag{A.25}
\]

For \( s^*_B(p) \leq s^*_B(p') \), condition (A.25) trivially holds for all \( p \geq v_H \). Suppose then that \( s^*_B(p) > s^*_B(p') \). By using \( \Pr(L|s_S) = 1 - \Pr(H|s_S) \), conditions (A.24) and (A.25) can be rewritten as:

\[
A \geq \Pr(H|s_S)[A + C] \tag{A.26}
\]

\[ S \] approaches \( \bar{s}_S \). On the other hand, the term \( [p - u_L]/[u_H - p] \) is bounded away from zero by \( [v_L - u_L]/[u_H - u_L] > 0 \) since \( S \) would surely lose from announcing a price below \( v_L \). Hence, \( f(s^*_B|H)/f(s^*_B|L) \) must also go to infinity which implies that \( s^*_B = \bar{s}_B \). Symmetrically, suppose that \( s_S = \bar{s}_S \). The ratio \( g(s_S|L)/g(s_S|H) \) converges to zero. For all \( p < u_H \), \( f(s^*_B|H)/f(s^*_B|L) \) must go to zero as well. But this implies \( s^*_B = \bar{s}_B \). \( \square \)
From (A.20), the minimum price at which she accepts to sell is

\[ A \geq \Pr(H|s_S)[A + C] \]  \hspace{1cm} (A.27)

where:

\[ A = [1 - F(s_B^*(p)|L)](p - v_L) - [1 - F(s_B^*(p')|L)](p' - v_L) \]  \hspace{1cm} (A.28)

\[ C = [1 - F(s_B^*(p')|H)](p' - v_H) - [1 - F(s_B^*(p)|H)](p - v_H) \]  \hspace{1cm} (A.29)

We distinguish between two cases: \( A < 0 \) (Case 1) and \( A \geq 0 \) (Case 2). **Case 1:** \( A < 0 \). If \( A \) is negative, then, from condition (A.26), \( A + C \) is also negative. One can then rewrite (A.24) and (A.25) respectively as:

\[ \Pr(H|s_S') \geq \frac{A}{A + C} \]  \hspace{1cm} (A.30)

\[ \Pr(H|s_S) > \frac{A}{A + C} \]  \hspace{1cm} (A.31)

Since, from the MLRP, \( \Pr(H|s_S') < \Pr(H|s_S) \), condition (A.31) is always verified when (A.30) holds. **Case 2:** \( A \geq 0 \). In this case, it is easy to verify that:

\[ \frac{1 - F(s_B^*(p)|L)}{1 - F(s_B^*(p')|L)} \geq \frac{p' - v_L}{p - v_L} > \frac{p' - v_H}{p - v_H} \]  \hspace{1cm} (A.32)

where the first inequality comes from \( A \geq 0 \) and the second from \( p > p' \). As shown in the proof of lemma 5, the MLRP implies that the following holds:

\[ \frac{1 - F(s_B^*(p)|H)}{1 - F(s_B^*(p')|H)} \geq \frac{1 - F(s_B^*(p)|L)}{1 - F(s_B^*(p')|L)} \]  \hspace{1cm} (A.33)

since \( s_B^*(p) > s_B^*(p') \). This, together with (A.32) implies \( C < 0 \). Given \( A \geq 0, C < 0, \) and \( \Pr(H|s_S) \leq 1, \) (A.27) is trivially verified.

**Lemma 10.** For any given \( s_B^* \), the minimum price at which \( S \) is willing to sell is an increasing function of \( s_S \) mapping \([s_S, \bar{s}_S]\) into the interval \([v_L, v_H]\).

**Proof.** From (A.20), the minimum price at which \( S \) accepts to trade is:

\[ v_H \Pr(H|s_S)[1 - F(s_B^*(p)|H)] + v_L(1 - \Pr(H|s_S))[1 - F(s_B^*(p)|L)] \]

\[ \frac{\Pr(H|s_S)[1 - F(s_B^*(p)|H)] + (1 - \Pr(H|s_S))[1 - F(s_B^*(p)|L)]}{\Pr(H|s_S)[1 - F(s_B^*(p)|H)] + (1 - \Pr(H|s_S))[1 - F(s_B^*(p)|L)]} \]  \hspace{1cm} (A.34)

This expression is increasing in \( \Pr(H|s_S) \). Seller \( S \)'s assessment of her quality upon observing \( s_S \) is consistent with Bayes rule:

\[ \Pr(H|s_S) = \frac{\lambda g(s_S|H)}{\lambda g(s_S|H) + (1 - \lambda)g(s_S|L)} \]  \hspace{1cm} (A.35)

From the MLRP, the above is an increasing function of \( s_S \). Hence, the minimum price is increasing in \( s_S \). Given that \( g(s_S|H)/g(s_S|L) \) has full support \((0, \infty)\) for \( s_S \in [s_S, \bar{s}_S] \), it is easy to verify that \( \Pr(H|s_S) \) converges to one (resp. zero) when \( s_S \) converges to \( \bar{s}_S \) (resp. \( s_S \)). Hence, when \( S \) observes \( \bar{s}_S \) (resp. \( s_S \)) the minimum price at which she accepts to sell is \( v_H \) (resp. \( v_L \)).

\[ \square \]
Lemma 11. There is no separating equilibrium in which trade occurs.

Proof. This follows from lemma 7. A separating equilibrium is a one-to-one map \( p(\cdot) \) from the set of realizations of \( S \)'s signal, \( s_S \), into the set of prices. Suppose then that there is a price \( p \) that is only announced by \( S \) when observing \( s_S \). Since \( s'_B \) would be equal to \( \bar{S}_B \), type \( s_S \) would be able to sell with probability zero. As a result, if \( s'_B < \bar{S}_B \) at any other price above \( v_L \), type \( s_S \) would always profit from mimicking. However, given lemma 10, \( s'_B < \bar{S}_S \) at some \( p > v_L \) is necessary for trade in equilibrium. Hence, full separation and trade cannot coexist.

Lemma 12. There is no NWBR-refined equilibrium in which trade only occurs at prices \( p \leq v_H \).

Proof. Assume that there is an equilibrium such that trade only occurs at prices lower than or equal to \( v_H \). Using the same argument introduced for lemma 11, it is easy to show that, in this equilibrium, \( S \) would never announce a price revealing its signal when observing \( s_S = \bar{S}_S \). In short, the realization \( \bar{S}_S \) must be pooled with other realizations. From lemma 10, this implies that the price at which \( S \) trades when observing \( \bar{S}_S \) must be above \( v_L \). As a result, in the candidate equilibrium, \( S \) makes positive expected profits when observing \( \bar{S}_S \). We show that this equilibrium violates NWBR. From lemma 10, the minimum price at which \( S \) is willing to trade for \( s_S = \bar{S}_S \) is \( v_H \). Hence, when observing \( \bar{S}_S \), \( S \) makes zero expected profits in equilibrium (if trade occurs at \( v_H \), she announces \( v_H \); otherwise, she announces a price at which no trade occurs – e.g. \( u_H \)). For this to be an equilibrium, at any out of equilibrium price \( p' \in (v_H, u_H) \), \( B \)'s threshold must be \( s'_B = \bar{S}_B \) (otherwise \( S \) would announce \( p' \) when observing \( \bar{S}_S \)). On the other hand, \( B \)'s threshold is equal to \( \bar{S}_B \) only if he believes that \( S \) has observed \( \bar{S}_S \). These beliefs however violate NWBR. To see this, notice that type (realization) \( \bar{S}_S \) would deviate to \( p' > v_H \) only if \( s'_B \) were low enough (since she makes positive profits in the candidate equilibrium), whereas type \( \bar{S}_S \) would deviate to \( p' \) for any \( s'_B < \bar{S}_B \) (since she makes zero profits in the candidate equilibrium). Hence, type \( \bar{S}_S \) can be eliminated according to NWBR.

Lemma 13. In any NWBR-refined equilibrium in which trade occurs there exists \( s'_S < \bar{S}_S \) such that \( S \) announces the same \( \bar{p} \in (v_H, u_H) \) for all realizations \( s_S \in \{s'_S, \bar{S}_S\} \).

Proof. We have already shown that there is no NWBR-refined equilibrium in which trade only occurs at prices lower than or equal to \( v_H \). Hence, trade must also occur at some price(s) greater than \( v_H \). In particular, the sorting condition ensures that \( S \) always announces a price \( \bar{p} > v_H \) when observing \( \bar{S}_S \). The sorting condition also implies that, if \( \bar{p} \) is the price announced by \( \bar{S}_S \), then it must be the maximum price announced in equilibrium. We first show that there exists a realization \( s'_S < \bar{S}_S \) for which \( S \) announces \( \bar{p} \). Then we show that \( S \) announces \( \bar{p} \) for all realizations \( s_S > s'_S \). Suppose then, by contradiction, that \( S \) announces \( \bar{p} \) only when observing \( \bar{S}_S \) so that \( \bar{p} \) perfectly reveals \( \bar{S}_S \). From lemma 7, \( s'_B = \bar{S}_B \). This implies that \( S \) is able to trade with probability one. But then, \( S \) would always profit from mimicking when observing lower signal realizations, given that \( \bar{p} \) is the maximum price at which trade can occur. Hence, there must be some realization \( s'_S < \bar{S}_S \) for which \( S \) also announces \( \bar{p} \), so that \( \bar{p} \) does not perfectly reveal \( \bar{S}_S \). If \( S \) announces \( \bar{p} \) for some \( s'_S \) and does not for some
From the previous result, any NWBR-refined equilibrium with trade implies that

\[ s'_S > s'_S, \]

then the sorting condition is violated. Hence, given any \( s'_S \) for which \( S \) announces \( \overline{p} \), \( S \) announces \( \overline{p} \) for all \( s_S \in [s'_S, \overline{s}_S] \).

\[ \square \]

**Lemma 14.** No equilibrium in which trade occurs survives NWBR.

**Proof.** From the previous result, any NWBR-refined equilibrium with trade implies that there is a set \( \Sigma(\overline{p}) = [s'_S, \overline{s}_S] \) of realizations for which \( S \) announces price \( \overline{p} \in (v_H, u_H) \). Consider a deviation \( p > \overline{p}, p < u_H \). We first show that all \( s_S < \overline{s}_S \) can be eliminated according to NWBR so long as they belong to \( \Sigma(\overline{p}) \). Then we show that also \( s_S < \overline{s}_S \) which do not belong to \( \Sigma(\overline{p}) \) can be eliminated. Finally, we use these results to show that there always exists \( p > \overline{p} \) such that the probability to sell at \( p \) is not smaller than the probability to sell at \( \overline{p} \) given \( B \)'s refined beliefs (which implies that it is always optimal to deviate to \( p \)).

Consider first types (realizations) \( s_S \in \Sigma(\overline{p}) \). Suppose that, upon observing the deviation \( p > \overline{p}, B \) uses threshold \( s_B^D \). Type \( s_S \) would (weakly) benefit from the deviation if:

\[
\Pr(H|s_S)[1 - F(s_B^D|H)](p - v_H) + \Pr(L|s_S)[1 - F(s_B^D|L)](p - v_L) \geq
\Pr(H|s_S)[1 - F(s^*_B|H)](\overline{p} - v_H) + \Pr(L|s_S)[1 - F(s^*_B|L)](\overline{p} - v_L) \tag{A.36}
\]

According to NWBR, type \( s_S \in \Sigma(\overline{p}) \) is eliminated if the set of all values for \( s_B^D \) that make him weakly benefit from the deviation is strictly contained in the set of values for \( s_B^D \) that make some other type strictly benefit. We can thus eliminate \( s_S \in \Sigma(\overline{p}) \) if, for all \( s_B^D \) such that (A.36) holds, a seller observing realization \( \overline{s}_S \) strictly benefits from the deviation. This happens if

\[
\Pr(H|\overline{s}_S)[1 - F(s_B^D|H)](p - v_H) + \Pr(L|\overline{s}_S)[1 - F(s_B^D|L)](p - v_L) >
\Pr(H|\overline{s}_S)[1 - F(s^*_B|H)](\overline{p} - v_H) + \Pr(L|\overline{s}_S)[1 - F(s^*_B|L)](\overline{p} - v_L) \tag{A.37}
\]

According to lemma 9 the above inequality is always verified given (A.36), since \( p > \overline{p} \) and \( \overline{s}_S > s_S \). Therefore, types \( s_S \in \Sigma(\overline{p}) \) can be eliminated.

We now show that all \( s_S \notin \Sigma(\overline{p}) \) can also be eliminated. Consider then \( s_S \notin \Sigma(\overline{p}) \). In equilibrium, \( s_S \) announces some price \( p' \) to which \( B \) replies with a threshold \( s'_B \). From incentive compatibility of type \( s_S \) it follows that:

\[
\Pr(H|s_S)[1 - F(s'_B|H)](p' - v_H) + \Pr(L|s_S)[1 - F(s'_B|L)](p' - v_L) \geq
\Pr(H|s_S)[1 - F(s^*_B|H)](\overline{p} - v_H) + \Pr(L|s_S)[1 - F(s^*_B|L)](\overline{p} - v_L) \tag{A.38}
\]

Type \( s_S \) weakly benefits from a deviation to \( p \) when:

\[
\Pr(H|s_S)[1 - F(s_B^D|H)](p - v_H) + \Pr(L|s_S)[1 - F(s_B^D|L)](p - v_L) \geq
\Pr(H|s_S)[1 - F(s'_B|H)](p' - v_H) + \Pr(L|s_S)[1 - F(s'_B|L)](p' - v_L) \tag{A.39}
\]

Hence, in order to eliminate \( s_S \), it is sufficient to show that \( \overline{s}_S \) strictly benefits for any \( s_B^D \) such that:

\[
\Pr(H|s_S)[1 - F(s_B^D|H)](p - v_H) + \Pr(L|s_S)[1 - F(s_B^D|L)](p - v_L) \geq
\Pr(H|s_S)[1 - F(s^*_B|H)](\overline{p} - v_H) + \Pr(L|s_S)[1 - F(s^*_B|L)](\overline{p} - v_L) \tag{A.40}
\]
but this, again, is implied by lemma 9 so that \( s_s \notin \Sigma(\tilde{p}) \), can be eliminated.

In summary, all realizations \( s_s < \bar{s}_s \) can be eliminated. It follows that \( B \), upon observing \( p \), should believe that the deviation comes from seller of type \( \bar{s}_s \). But then, upon observing \( p \), \( s_B^* = \bar{s}_B \). Hence, \( S \) would be able to sell with probability one and, as a result, would always benefit from deviating to \( p \). □

**Lemma 15.** There is always a NWBR-refined equilibrium in which trade does not occur.

**Proof.** Consider a situation in which \( S \) always announces \( p = u_H \) and \( B \) never buys (i.e. \( s_B^* = \bar{s}_B \)). Out of equilibrium beliefs are such that, upon observing any deviation, \( B \) believes that \( S \) has received signal \( s_S \) so that \( s_B^* = \bar{s}_B \) and \( B \) buys with probability zero upon observing any deviation \( p < u_H \). This is clearly an equilibrium. We now show that it is also robust to NWBR. Given any \( p > v_L \), type \( s_S \) is willing to deviate to \( p \) whenever \( s_B^* < \bar{s}_B \). As a result, for all \( s_S > s_S \), the set of threshold values that make type \( s_S \) willing to deviate is never contained in the set of threshold values that make \( s_S \) willing to deviate. □

### A.3 Proof of Proposition 3

In the game in which \( S \) sets the price, payoffs were given by:

<table>
<thead>
<tr>
<th>( q )</th>
<th>Price Taker (( B ))</th>
<th>Price Setter (( S ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{H} )</td>
<td>( u_H - p )</td>
<td>( p - v_H )</td>
</tr>
<tr>
<td>( \tilde{L} )</td>
<td>( u_L - p )</td>
<td>( p - v_L )</td>
</tr>
</tbody>
</table>

with \( u_H > u_L \), \( v_H > v_L \), \( u_H > \tilde{v}_H \), and \( v_L > \tilde{u}_L \). Consider now the game in which \( B \) announces a price \( p \) and gains from trade are reversed: \( v_H > u_H \) and \( u_L > v_L \). In words, quality \( \tilde{H} \) generates negative gains from trade while quality \( \tilde{L} \) yields positive gains. We want to show that the two games are the same. Let \( A \in \mathbb{R} \) be a number greater than \( v_H \). Redefine \( B \)'s action as \( \hat{p} = A - p \) and qualities as \( \tilde{H} = L \), and \( \tilde{L} = H \). Redefine also the valuations as follows: \( \hat{v}_H = A - u_L \), \( \hat{v}_L = A - u_H \), \( \hat{u}_H = A - v_L \), \( \hat{u}_L = A - v_H \). Clearly, the conditions \( \tilde{u}_H > \tilde{u}_L \), \( \tilde{v}_H > \tilde{v}_L \), \( \tilde{u}_H > \tilde{v}_H \), and \( \tilde{v}_L > \tilde{u}_L \) hold. Also, it is easy to check that net payoffs are unaffected by these transformations. The net payoffs for the game in which \( B \) sets the price can thus be expressed as:

<table>
<thead>
<tr>
<th>( q )</th>
<th>Price Taker (( S ))</th>
<th>Price Setter (( B ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{H} )</td>
<td>( \hat{u}_H - \hat{p} )</td>
<td>( \hat{p} - \hat{v}_H )</td>
</tr>
<tr>
<td>( \tilde{L} )</td>
<td>( \hat{u}_L - \hat{p} )</td>
<td>( \hat{p} - \hat{v}_L )</td>
</tr>
</tbody>
</table>

Hence, this is the same game as that in which \( S \) sets the price. □

### A.4 Proof of Proposition 4.

In what follows, we ignore \( S \)'s interim participation constraint by assuming \( c < u_H - v_H \). This is however only for expositional purposes and it is easy to verify that proposition 4 would not change if this assumption were relaxed. Notice also that, since \( S \) can choose not to put the good on offer at time 2, whenever \( S \) sticks to \( p_0 \) in an
equilibrium with trade, then \( p_0 \geq v_H \). This follows since type \( H \) would never put the good on offer at lower prices and type \( L \) would never be able to trade unless pooled with type \( H \). We start off by showing that there is always a degenerate equilibrium robust to NWBR (Lemma 16). We then restrict attention to non-degenerate equilibria. We first analyze the sorting condition (Lemma 17). Then, given \( p_2 \), we determine \( B \)'s best reply conditional on \( B \)'s information at stage \( t = 3 \) (Lemma 18). Lemma 19 provides, for a given \( p_0 \), necessary conditions for equilibria with trade in the signaling game starting at \( t = 1 \). Lemma 20 characterizes the NWBR-refined equilibria in the signaling game starting at \( t = 1 \). Given the non-degenerate NWBR-refined equilibrium in the signaling game, lemma 21 reduces \( S \)'s problem in choosing the optimal level of \( p_0 \) to a constrained optimization problem. Finally, we prove the results stated in the proposition.

**Lemma 16.** There always exists a degenerate NWBR-refined equilibrium of the entire game in which trade never occurs.

**Proof.** Consider the following degenerate equilibrium. Suppose that, for any \( p_0 \), \( S \) always announces \( p_2 = u_H \) at \( t = 2 \) and \( B \) never buys at any price. This is clearly an equilibrium if beliefs assign probability one to any deviation \( p < u_H \) (including sticking to \( p_0 \)) emanating from type \( L \). These beliefs do not contradict NWBR since, for any \( p \in (v_H, u_H) \), both types strictly benefit whenever the probability of trade is positive. (For \( p = v_H \) only type \( L \) strictly benefits). Hence, type \( L \) cannot be eliminated. \( \square \)

**Lemma 17.** (Sorting with switching cost) Suppose that \( B \) follows a threshold strategy on her signal. Then: a) Consider prices \( p_2 \neq p_0 \) and \( p_2 \neq p_0 \) announced at \( t = 2 \). For \( p_2 > p_2 \) (resp. \( p_2 < p_2 \)\), if type \( L \) (resp. \( H \)) weakly prefers \( p_2 \) to \( p_2 \), then type \( H \) (resp. \( L \)) strictly prefers \( p_2 \). b) Consider prices \( p_2 = p_0 \) and \( p_2 \neq p_0 \) announced at \( t = 2 \). For \( p_2 > p_0 + c \) (resp. \( p_2 < p_0 + c \)), if type \( L \) (resp. \( H \)) weakly prefers to switch to \( p_2 \) rather than sticking to \( p_2 = p_0 \), then type \( H \) (resp. \( L \)) strictly prefers to switch to \( p_2 \).

**Proof.** We prove the result for case b). The proof for case a) is very similar and is therefore omitted. Let \( s_0 \) be the threshold on \( B \)'s signal when observing \( p_2 = p_0 \) and let \( s_0' \) denote the threshold when \( B \) observes \( p_2 > p_0 + c \). Type \( L \) weakly benefits from the deviation whenever:

\[
[1 - F(s_0' | L)](p_2 - c - v_L) \geq [1 - F(s_0 | L)](p_0 - v_L)
\]  

(A.42)

Type \( H \) strictly benefits if:

\[
[1 - F(s_0' | H)](p_2 - c - v_H) > [1 - F(s_0 | H)](p_0 - v_H)
\]  

(A.43)

Given \( p_2 > p_0 + c \), both types strictly benefit whenever \( s_0' \leq s_0 \). Hence, we can focus on the case in which \( s_0' > s_0 \). Given \( s_0' > s_0 \), the MLRP implies:

\[
\frac{1 - F(s_0' | H)}{1 - F(s_0 | H)} \geq \frac{1 - F(s_0' | L)}{1 - F(s_0 | L)}
\]  

(A.44)

(this has already been proved in the proof of lemma 5). This implies that type \( H \) strictly benefits whenever type \( L \) weakly benefits if:

\[
\frac{p_0 - v_H}{p_2 - c - v_L} > \frac{p_0 - v_H}{p_2 - c - v_H}
\]  

(A.45)

34
which is verified for $p_2 > p_0 + c$. Hence, type $H$ strictly benefits whenever type $L$ weakly benefits. Assume now that $p_2 < p_0 + c$. In this case both types strictly lose whenever $s'_2 \geq s_0$. Hence, we can focus on the case $s'_2 < s_0$. Here, the MLRP implies:

$$\frac{1 - F(s'_2|H)}{1 - F(s_0|H)} \leq \frac{1 - F(s'_2|L)}{1 - F(s_0|L)}$$ \quad (A.46)

A symmetric argument to the one just given then shows that type $L$ strictly benefits when type $H$ weakly benefits provided that $p_2 < p_0 + c$.  

**Lemma 18.** Assume that, at stage $t = 2$, both type $H$ and type $L$ announce the same price $p_2$ with probabilities $\beta_H \in (0, 1]$ and $\beta_L \in (0, 1]$ respectively. Then the following holds: a) upon observing $p_2$, $B$ uses a threshold $s^*$ which depends on $p_2$; b) $B$ buys with positive probability (i.e. $s^* < \bar{s}$) if and only if $p_2 < u_H$; c) in a pure pooling (i.e. with $\beta_L = \beta_H = 1$):

$$f(s|H) \geq \frac{1 - \lambda}{\lambda} \frac{p_2 - u_L}{u_H - p_2} f(s|L)$$ \quad (A.47)

for any $s \geq s^*$.

**Proof.** Assume that, at $t = 2$, $S$ chooses a price $p_2$. $B$, who receives a signal $s$, chooses to buy at $p_2$ if:

$$\frac{\beta_H \lambda f(s|H)}{\beta_H \lambda f(s|H) + \beta_L (1 - \lambda) f(s|L)} u_H + \frac{\beta_L (1 - \lambda) f(s|L)}{\beta_H \lambda f(s|H) + \beta_L (1 - \lambda) f(s|L)} u_L - p_2 \geq 0$$ \quad (A.48)

Given monotonicity, $B$ uses a threshold strategy on his own signal: he chooses to buy at $p_2$ if his signal $s$ is above a threshold $s^*$, where $s^*$ solves:

$$\frac{f(s^*|H)}{f(s^*|L)} = \frac{\beta_L (1 - \lambda) p_2 - u_L}{\beta_H \lambda u_H - p_2}$$ \quad (A.49)

Given $f(|H)|/f(|L)|$ increasing and with full support $(0, \infty)$, there exists $s^* < \bar{s}$ satisfying (A.49), provided $p_2 < u_H$. Since $s^*(p_2) < \bar{s}$, trade occurs with positive probability. Finally, solving (A.48) for $f(s|H)$ and setting $\beta_L = \beta_H = 1$ yields (A.47).  

**Lemma 19.** For a given $p_0$, a PBE of the signaling game starting at $t = 1$ is characterized by a positive probability of trade only if it is either: a) a pooling equilibrium in which both types announce the same price $p_2 < u_H$, or b) when $p_0 < u_H - c$, a hybrid in which both types randomize between $p_2 = p_0$ and $p_2' = p_0 + c$.

**Proof.** A PBE can be a separating, a pooling, or a hybrid. In a separating equilibrium, type $L$ would not be able to trade and would make zero profits. Given $u_H > u_L$, whenever type $H$ profits from trade, type $L$ would profit from mimicking. Hence, no separating equilibrium with trade is possible. Consider now a hybrid equilibrium in which either or both types randomize at $t = 2$. Let $P_q$ be the set of prices announced by type $q$ at $t = 2$. Notice that $P_L \subseteq P_H$. Using lemma 17, it is immediate to show that the set $P_H$ contains at most two elements. Denote them with $p_2$ and $p_2'$. Then, either $P_H = \{p_2, p_2'\}$ while $P_L \subset P_H$, or both types announce both prices with
positive probability: \( P_L = P_H \). The first case is clearly not incentive compatible given that type \( H \) would be able to sell with probability one: either type \( L \) would mimic or type \( H \) would find it optimal not to announce the price not announced by type \( L \). Consider then the case \( P_L = P_H \) in which both types announce both prices with positive probability. If both \( p_2 \) and \( p_2' \) are different from \( p_0 \), then the equilibrium violates lemma 17: either of the two types would always strictly prefer to announce either of the two prices. Hence, randomization is not possible. (The proof of lemma 18 has already shown that \( B \) follows a threshold strategy on her signal when the price is announced by both types with positive probability). If one price, say \( p_2 \), is equal to \( p_0 \) but \( p_2' \neq p_0 + c \) then, again, the equilibrium violates lemma 17. As a result, candidate equilibria in which trade occurs must take either of the following forms. They can be pooling equilibria in which both types announce only one price (which must be less than \( u_H \) to induce \( B \) to buy with positive probability). Alternatively, they can be hybrid equilibria in which both types randomize between \( p_0 \) and \( p_0 + c \). \( \square \)

**Lemma 20.** Consider the signaling game starting at \( t = 1 \). If and only if \( p_0 \in [u_H - c, u_H) \), there exists a NWBR-refined equilibrium in which trade occurs with positive probability. This is a pooling characterized by \( p_2 = p_0 \) (no switching).

**Proof.** According to NWBR, type \( q \in \{L, H\} \) can be eliminated for a deviation \( p \) if the set of \( B \)'s threshold values for which type \( q' \) strictly benefits from the deviation strictly includes the set of \( B \)'s threshold values for which type \( q \) weakly benefits. We start by showing that equilibria with trade which involve switching fail NWBR. Consider the case of a pooling with \( p_2 \neq p_0 \). Assume that \( p_2 < u_H \) (which is necessary for trade). Then, lemma 17 implies that type \( L \) is eliminated for any deviation \( p > p_2 \). Hence, \( S \) would be able to sell with probability one so long as \( p < u_H \). Since there exists \( p \in (p_2, u_H) \) at which \( S \) would clearly profit, all \( p_2 < u_H \) fail NWBR. Only \( p_2 = u_H \), which involves no trade, passes NWBR. Consider now the case of a pooling with no switching: \( p_2 = p_0 \). Given a deviation \( p > p_2 + c \), lemma 17 implies that \( B \) should assign probability zero to the deviation emanating from \( L \). Hence, in any pooling equilibrium with \( p_2 < u_H - c \), \( S \) could profit from deviating to some \( p \) such that \( u_H > p > p_2 + c \). So long as \( p < u_H \), \( S \) would be able to sell at a higher (net of \( c \)) price with probability one. As a result, among the pooling equilibria with no switching, only equilibria with \( p_2 \in [u_H - c, u_H] \) pass NWBR. Notice that this argument also eliminates the hybrid equilibrium in which \( S \) randomizes between \( p_0 \) and \( p_0 + c \).\footnote{Unless \( p_0 + c = u_H \), the hybrid equilibrium would fail NWBR. However, if \( p_0 + c = u_H \), no trade would occur at \( p_0 + c \) if both types announce \( p_0 + c \) with positive probability. In this case, \( S \) cannot be indifferent between \( p_0 \) and \( p_0 + c \). If only type \( H \) announces \( p_0 + c \) with positive probability and \( B \) randomizes when observing \( p_0 + c \), type \( L \) would benefit from announcing \( p_0 + c \) whenever the probability to sell at \( p_0 + c \) is such that type \( H \) is indifferent between \( p_0 \) and \( p_0 + c \).}

We still have to show that trade occurs with positive probability in these equilibria. This follows from lemma 18. (For any \( p_2 < u_H \), there is always a positive probability of trade). In summary, NWBR-refined equilibria with trade must be such that: a) \( p_2 \in [u_H - c, u_H) \); b) \( p_2 = p_0 \). \( \square \)

**Lemma 21.** Let \( p_0 \) be the price chosen by \( S \) at \( t = 0 \). A NWBR-refined equilibrium of the entire game in which trade occurs with positive probability exists and is a pooling
equilibrium in which: a) \( p_2 = p_0 \); b) \( p_0 \) solves:

\[
\max_p \lambda [1 - F(s^*(p)|H)](p - v_H) + (1 - \lambda)[1 - F(s^*(p)|L](p - v_L)
\]

\[
s.t. \quad u_H - c \leq p \leq u_H \quad (A.50)
\]

**Proof.** Problem (A.50) maximizes \( S \)'s expected profits under the constraint \( u_H - c \leq p \leq u_H \). Restrict attention to the NWBR-refined equilibrium of the signaling game starting at \( t = 1 \). For \( S \)'s problem to be expressed as the constrained optimization problem (A.50), it is sufficient that, for any \( p_0 \in [0, u_H - c] \), \( S \)'s profits are less than or equal to the profits she could make by setting some \( p_0 \in [u_H - c, u_H] \). One can invoke lemma 20 to prove the result. When \( p_0 \in [0, u_H - c] \), no trade occurs and \( S \) makes zero profits. This outcome, however, can always be achieved by choosing \( p_0 = u_H \). If \( p_0 \in [u_H - c, u_H] \) then the prevailing equilibrium is such that trade occurs with positive probability in a pooling characterized by \( p_2 = p_0 \) (lemma 20). Given \( p_2 \geq u_H - c > v_H \), \( S \) always makes positive profits when trade occurs. Hence, at \( t = 0 \), \( S \)'s problem can be written as (A.50). As for existence, this follows from continuity of the objective function and the compactness of \([u_H - c, u_H]\). \( \square \)

We are now ready to prove the main result:

**Proof of Proposition 4.**

The game in which \( c = 0 \) is identical, from \( t = 1 \) onward, to the game considered in proposition 2. Hence, no trade is possible. For \( c > 0 \), lemma 16 implies that there is always a (degenerate) equilibrium characterized by no trade. From lemma 21, when \( c > 0 \), there is also a pooling equilibrium which is non-degenerate in which \( p_2 = p_0 \). We start by showing that in this equilibrium the probability of trade and \( S \)'s profits are positive. At \( t = 0 \), \( S \) chooses \( p_0 \in [u_H - c, u_H] \) as to solve (A.50). For any \( p_0 < u_H \), lemma 18 ensures that trade occurs with positive probability. Given \( p_0 \geq u_H - c > v_H \), expected profits are positive unless the amount of trade is zero. Hence, \( p_0 = u_H \) is never optimal. We now show that expected total surplus is also bounded away from zero. Total surplus is:

\[
\lambda[u_H - v_H][1 - F(s^*|H)] + (1 - \lambda)[u_L - v_L][1 - F(s^*|L)] \quad (A.51)
\]

where the second term is negative. Total surplus is positive when:

\[
[1 - F(s^*|H)] > \frac{(1 - \lambda)[v_L - u_L]}{\lambda[u_H - v_H]}[1 - F(s^*|L)] \quad (A.52)
\]

Since inequality (A.47) must hold for every \( s > s^* \), one can integrate both sides of the inequality between \( s^* \) and \( \bar{s} \):

\[
[1 - F(s^*|H)] \geq \frac{1 - \lambda}{\lambda} \frac{p_0 - u_L}{u_H - p_0}[1 - F(s^*|L)] \quad (A.53)
\]

Given (A.53), condition (A.52) always holds for \( p_0 \in [u_H - c, u_H] \). We now turn attention to the comparative statics result. This follows from a standard argument. Notice that, in problem (A.50), \( c \) appears only in the constraint. Suppose that when
$c = \hat{c}$ the solution of $S$'s problem is $p = p_0$. If $c$ increases to some $\tilde{c} > \hat{c}$, $S$ always makes at least the same profits as with $\hat{c}$ since she can still choose $p_0$. Hence, profits are weakly increasing in $c$. Note that an increase in $c$ implies that the optimal price $p_0$ either decreases or stays constant. Hence, given the weak increase in expected profits, the probability of trade must weakly increase. Also, since $p_0$ weakly decreases, and the same pool of quality offered at $p_0$ stays the same, $B$'s expected surplus weakly increases. This implies that total expected surplus from trade weakly increases. $\Box$
References


