Should I Stay or Should I Go?

Migration under Uncertainty: A New Approach

by

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Abstract

This paper considers migration as an investment decision. We develop a continuous-time stochastic model to explain the optimal timing of migration, in the presence of ongoing uncertainty over wage differentials. Our results reveal that households prefer to wait before migrating, even if the present value of the wage differential is positive, because of the uncertainty and the sunk costs associated with migration. An increased degree of risk aversion discourages migration, and interacts with the other variables and parameters affecting migration by exacerbating their effects. Households are less likely to migrate into rural areas with a less predictable income profile.

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1. Introduction

People migrate in order to increase their welfare. The seminal papers on migration (Lewis, 1954; Todaro, 1969, 1976; Harris and Todaro, 1970) assume that the option to migrate for the household is limited to the exact point in time when the present value of the expected wage differential becomes positive. Although income can be an important signal to migrate, there are three issues that need attention. First, if households fail to take up the option to migrate at the moment in time when it is considered optimal to do so, then this is assumed to be lost forever. This means that for any household there is a now-or-never approach to migration that is not only restricting, but also implies that households cannot contemplate migration as a welfare-augmenting strategy at any point in the future if they do not migrate today. This amounts to a penalty that households must incur if they exercise caution and so seems counter-rational. Second, standard migration theory predicts that out-migration is a rational response to a positive expected wage differential, but household behaviour does not always conform to this prediction. Third, migration has continued even in the absence of a wage differential. Migration has not been the equilibrating mechanism as argued by Lewis (1954) and Todaro (1969).

Oded Stark (1984, 1991; see also Stark and Bloom, 1985) addresses some of the issues raised by more conventional theories of migration by applying the notion of relative deprivation to households in the village of origin. The relative deprivation of a household is measured in terms of an income statistic other than their own which propels out-migration. Those households that earn an income higher than this income statistic feel no sense of relative deprivation and have no reason to move. However, those households earning less than the reference statistic do feel relatively deprived and have a propensity to migrate. Households whose income lies furthest away from the reference statistic feel more relatively deprived than those whose income is just less.

Relative deprivation permits an analysis of some observed migration patterns. Households continue migrating into urban areas even in the absence of a positive wage differential (see Fields, 1982; Schultz, 1982). The migration decision in the relative deprivation theory is motivated solely by reference to an income statistic in the area of origin rather than in the destination area. Migration has been observed to be highest in villages where the distribution of income is highly skewed (see the evidence cited in Stark, 1984). In such villages the number of those who feel relatively deprived will be high particularly if the income statistic they respond to is the average village income. Equally, in very poor villages the number of those households that feel relatively deprived will be small as average income will be low. Thus there is less incentive to migrate to diminish their sense of relative deprivation.

The principal aim of this paper is to set out a model of migration as a form of household investment under uncertainty. Although migration does not represent investment into a fixed asset, there is an inter-temporal trade-off. A sacrifice is made today in the expectation of future rewards. Lewis and Todaro considered migration behaviour in a static context. Whilst their contribution has provided a valuable starting point, the predictions of their theoretical approach have not been able to explain some puzzling aspects of migration behaviour. Stark (1991) has offered a different and novel approach to migration, but along with more conventional theorists there is no discussion of the irreversibility of the migration decision. Migration is not costless; it involves sunk costs which cannot be recouped at a later date, even though the act of migration itself can be reversed. Consideration of these costs is critical to the migration decision and indeed is consistent with rational behaviour.

Fundamentally, the decision to migrate must balance the expected future value of the wage differential against the level of sunk costs. The former must more than offset the latter: it is not sufficient for future expected earnings differentials to be merely positive. Household perceptions and expectations play an important role in the timing of migration. Since there is uncertainty associated with the wage differential, households incorporate this uncertainty in their decision to migrate. If households expect the future wage differential to rise above its current level, then they may choose to delay migration into the future. There can therefore be a value to waiting. The expectations of the future trend of the wage differential are a function of the information set available to the household. By contrast, standard theory tends to assume a static model and static expectations.

The analysis of migration as an investment under uncertainty allows for a different interpretation of certain aspects of migration behaviour. Whilst a positive wage differential can be a signal to the household that migration may be an optimal strategy, the sunk costs associated with migration must be passed in order for migration to be desirable. This threshold is a function of the expected future wage differential, of the sunk costs and of the uncertainty. Increased uncertainty over the wage differential is likely to raise the threshold level at which migration becomes a desirable strategy. Increased uncertainty, therefore, has the effect of increasing the region of inertia whereby the household delays migration. Households exercise caution and delay migration if the wage differential is perceived to be volatile in the future. This is entirely rational, as increased expected household income is only desirable if accompanied by relative stability of this income.

Perceptions of the uncertainty over wages may explain why migration could occur given a wage differential that is small or even zero. Increased uncertainty associated with the wage in the village of origin relative to the destination wage may prompt out-migration even if the wage differential is zero. For risk averse households, reduced uncertainty is linked to increases in welfare and the decision to migrate will be brought forward. Schultz (1982) and Fields (1982) show that the elasticity of migration with respect to income in the village of origin is weak. This suggests that income by itself is insufficient to motivate migration, and that other factors, including possibly uncertainty, come into play.

Although the costs associated with migration are irreversible, migration itself can often be reversed. Households can decide to return to their village of origin. This option to return migrate can influence the initial migration decision. If households know that return is possible, then they might be more willing to undertake migration in the first place.

As a departure from the majority of studies on migration, the model presented in this paper concentrates on rural-rural migration. As rural-rural migration encompasses many different elements of migration patterns, rural-rural migration hereafter will refer to inter-state and/or intra-state flows. It will be assumed that migration in this case is longer term but not necessarily permanent, and thus it may also incorporate seasonal migration. However, the model can be adapted to consider the case of rural-urban migration by incorporating an asymmetry in the wage trends of rural and urban areas.

The structure of this paper is as follows. In section 2 the main theoretical model is presented. In section 3 the model is extended to consider risk averse households. Section 4 analyses migration under a neutral spread of the wage differential. Section 5 summarises the main results.

2. A theoretical model

In this section a continuous-time model of migration is developed where the wage differential evolves over time in a stochastic manner, and where uncertainty is never fully resolved. As the aim of this paper is to analyse rural-rural migration, it is appropriate, therefore, to assume symmetry in the wage profile between the area of origin and the area of destination. Specifically, the optimal decision rule for a household in the village of origin to migrate to a rural destination area is derived and then the optimal rule for a household in the destination area to return-migrate to the village of origin.

The analysis of migration presented examines both the cost of the initial migration and the cost of return migration. These costs are sunk and irreversible. It is established that both costs are relevant when a household considers its migration decision. There exists a region of inertia over which the household is unwilling to change the *status quo*. In other words, there is a range of values of the wage differential over which it is not optimal for the household to undertake migration in either direction. It is demonstrated that increases in the sunk costs widen the region of inertia while decreases have the opposite effect. This inertia results in a hysteresis effect. The optimal strategy thus depends on the past migration history of that household.

A rural household will not undertake out-migration at the point where the net present value (*NPV*) of the wage differential is positive and just sufficient to cover the costs as predicted by standard Marshallian microeconomic analysis. Rather, migration will only occur when the *NPV* is sufficiently high to compensate for the irreversibility of the migration decision. This will require a higher positive wage differential than prescribed in conventional migration theory.

A similar argument holds for return migration. For this to be the optimal strategy for a rural household, the wage differential must be negative and large in absolute value.

Let W^O be the wage in the village of origin, W^D the destination wage, and define $V = e^{W^D - W^O}$. Let I be the cost of initial migration, and E the cost of return migration. The stochastic structure of the model is as follows. The exponential of the wage differential, V, is assumed to follow a geometric Wiener process:

(1) $$dV = s_v V dz_v$$

This formulation of uncertainty as a geometric process implies that dV is proportional to the existing level of the wage differential V, rather than independent of it. Furthermore, it excludes the possibility that the stochastic process for the wage differential might have the origin as an absorbing state. In economic terms, this means that there can be negative values of the wage differentials, and that zero is not an absorbing state.

The stochastic process (1) implies that today's wage differential is the best predictor for tomorrow's wage differential. Hence, the trend component in wages is the same for both the village of origin and the village of destination. There is no fundamental asymmetry in wage trends between rural areas (in contrast to rural and urban areas). The component dz_v in (1) is a Wiener disturbance, which is defined as:

(2) $$dz_v(t) = e_v(t) \cdot \sqrt{dt}$$

where $e_v(t) \sim N(0,1)$ is a white noise stochastic process (see Cox and Miller, 1965). The Wiener component dz_v is therefore normally distributed with zero expected value and variance equal to dt:

(3) $$dz_v \sim N(0, dt)$$

In the present model it is assumed that there is no uncertainty over I and E [1]. When deriving the optimal behavioural rule for migration, it is necessary to distinguish whether the household is:

(a) in the village of origin;

(b) in the village of destination.

Two problems can, therefore, be identified:

Problem (a). Optimal migration rule for a household in the village of origin.

Problem (b). Optimal migration rule for a household in the village of destination.

Problem (a).

Bellman's dynamic programming approach is employed to obtain the optimal rule for migration. The household will remain in the village of origin as long as the wage differential is less than a critical value, and will migrate as soon as this critical value is reached. Note that the definition $V = e^{W^D - W^O}$ implies that the relevant range for the Bellman equation is of the form $V \in (0, V^H)$. As the wage differential tends to minus infinity, V approaches zero. The household will remain in the village of origin for $V < V^H$, and will migrate when $V = V^H$. Thus V^H represents the upper bound of the region of inertia in the migration behaviour of the household.

Let F^O be the measure of the value to the household in the village of origin of having the migration opportunity:

[1] In Khwaja (2000a) uncertainty over I and E are considered

(4) $$F^O = F^O(e^{W^D - W^O}) = F^O(V) \qquad V \in (0, V^H)$$

The Bellman equation (or the asset equation) for the dynamic programme of the household is:

(5) $$rF^O(V)dt = W^O dt + E[dF^O(V)]$$

or:

(5a) $$rF^O dt = W^O dt + E(dF^O)$$

where r is the instantaneous rate of interest. Formally, (5) or (5a) are obtained by equating the product of the rate of interest and the value of the asset (LHS) with the sum of the instantaneous benefit and the expected capital gain or loss from the asset (RHS).

The Bellman equation (5a) can be expanded by using Itô's Lemma of stochastic calculus (Dixit and Pindyck, 1994):

(6) $$dF^o = \frac{\partial F^O}{\partial t}dt + \frac{\partial F^O}{\partial V}dV + \frac{1}{2}\frac{\partial^2 F^O}{\partial V^2}(dV)^2$$

$$= \frac{1}{2}\sigma_v^2 V^2 F_{vv}^O dt + \sigma_v V F_v^O \varepsilon_v^2 dz_v$$

By taking expectations of (6) using (3) we have:

(7) $$E(dF^{O}) = \frac{1}{2}\sigma_{v}^{2}V^{2}F_{vv}^{O}dt$$

Replacing (7) into (5a) the Bellman equation is obtained as:

(8) $$rF^{O}dt = W^{O}dt + \frac{1}{2}\sigma_{v}^{2}V^{2}F_{vv}^{O}dt$$

Dividing (8) by dt a 2nd-order differential equation in $F^{O}(V)$ is obtained:

(9) $$\frac{1}{2}\sigma_{v}^{2}V^{2}F_{vv}^{O} - rF^{O} = -W^{O}$$

The solution to (9) is given by the sum of the general solution for the homogeneous equation and of a particular solution for the inhomogeneous equation. Therefore, a solution for the homogeneous equation must first be found:

(10) $$\frac{1}{2}\sigma_{v}^{2}V^{2}F_{vv}^{O} - rF^{O} = 0$$

Using a guess solution of the form:

(11) $$F^{O} = AV^{b}$$

implies

(11a) $$F_{v}^{O} = bAV^{b-1}$$

(11b) $$F^{O}_{vv} = b(b-1)AV^{b-2}$$

Substituting (11) and (11b) into the homogeneous equation (10) gives:

(12) $$\frac{1}{2}s_{v}^{2}b(b-1)AV^{b} - rAV^{b} = 0$$

Dividing (12) by AV^{b} leads to:

(13) $$\frac{1}{2}s_{v}^{2}b(b-1) - r = 0$$

The roots of the quadratic equation (13) are:

(14a) $$b_{1} = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2r}{s_{v}^{2}}} > 1$$

(14b) $$b_{2} = \frac{1}{2} - \sqrt{\frac{1}{4} + \frac{2r}{s_{v}^{2}}} < 0$$

The general solution for the homogeneous equation (10) is:

(15) $$F^{O}(V) = A_{1}V^{b_{1}} + A_{2}V^{b_{2}}$$

A particular solution for the inhomogeneous equation (9) takes the form:

(16) $$F^{O}(V) = K$$

where K is a constant. Replacing (16) into the differential equation (9) gives:

(17) $$K = \frac{W^O}{r}$$

The general solution for the second-order inhomogeneous differential equation (9) is given by:

(18) $$F^O(V) = A_1 V^{b_1} + A_2 V^{b_2} + \frac{W^O}{r} \qquad V \in (0, V^H)$$

Consider A_2. As $V \to 0$, $W^D - W^O \to -\infty$ and therefore the option to migrate should be worthless. Since $b_2 < 0$, in order to avoid $F^O(V) \to \infty$ as $V \to 0$, A_2 must be set to equal zero, i.e. A_2=0. Therefore, $F^O(V)$ is defined as :

(18') $$F^O(V) = A_1 V^{b_1} + \frac{W^O}{r} \qquad V \in (0, V^H)$$

Problem (b).

For a household in a rural destination village the Bellman equation is defined over the range $V \in (V^L, +\infty)$. The household will remain in the destination village as long as the wage differential is greater than a critical value, $\ln V^L$. The household will return to the village of origin only when V=V^L. The Bellman equation is:

(19) $$rF^{D}dt = W^{D}dt + E(dF^{D})$$

Proceeding as for Problem (a), a general solution of the form:

(20) $$F^{D}(V) = C_1 V^{b_1} + C_2 V^{b_2} + \frac{W^{D}}{r} \qquad V \in (V^{L}, +\infty)$$

is obtained.

Consider C_1. As $V \to \infty$, $W^{D} - W^{O} \to \infty$ and therefore the option to return migrate should be worthless. Since $b_1 > 1$, to avoid $F^{D}(V) \to \infty$ as $V \to \infty$, C_1 must be set to equal zero i.e., $C_1=0$. Therefore the general solution for F^D is:

(20') $$F^{D}(V) = C_2 V^{b_2} + \frac{W^{D}}{r} \qquad V \in (V^{L}, +\infty)$$

To determine A_1 and C_2 we use the value matching and smooth pasting conditions are used. The value matching conditions equate the values of the alternative options, open to the decision maker at each critical boundary. The smooth pasting conditions equate the marginal changes of the option values, at each one the critical boundaries (see Dixit and Pindyck, 1994). The value matching conditions for the problems indicated above are:

(21) $$F^{O}(V^{H}) = F^{D}(V^{H}) - I$$

(22) $$F^{D}(V^{L}) = F^{O}(V^{L}) - E$$

Equation (21) says that, given V^H, a household in the village of origin must be indifferent between remaining in the village and migrating to a destination village, whereby it will incur a cost I. Equation (22) says that, given V_L, a household in the destination village must be indifferent between remaining in the village and return migrating, whereby it will incur a cost E.

The smooth pasting conditions are:

(23) $$F_v^O(V^H) = F_v^D(V^H)$$

(24) $$F_v^D(V^L) = F_v^O(V^L)$$

Equations (23) and (24) say that, at the critical boundaries, the value functions for the household in the village of origin and for the household in the village of destination must be tangential to each other.

By using (18') and (20'):

(25) $$F^O(V) = A_1 V^{b_1} + \frac{W^O}{r}$$

is obtained.

(26) $$F^D(V) = C_2 V^{b_2} + \frac{W^D}{r} = C_2 V^{b_2} + \frac{\ln V}{r} + \frac{W^O}{r}$$

(27) $$F_v^O(V) = A_1 b_1 V^{b_1 - 1}$$

(28) $$F_v^D(V) = C_2 b_2 V^{b_2 - 1} + \frac{1}{rV}$$

By replacing (25)-(28) into (21)-(24) the following system of equations for A_1, C_2, V^L and V^H is obtained:

(29) $$A_1 V^{Hb_1} + \frac{W^O}{r} = C_2 V^{Hb_2} + \frac{\ln V^H}{r} + \frac{W^O}{r} - I$$

(30) $$C_2 V^{Lb_2} + \frac{\ln V^L}{r} + \frac{W^O}{r} = A_1 V^{Lb_1} + \frac{W^O}{r} - E$$

(31) $$A_1 b_1 V^{Hb_1 - 1} = C_2 b_2 V^{Hb_2 - 1} + \frac{1}{rV^H}$$

(32) $$A_1 b_1 V^{Lb_1 - 1} = C_2 b_2 V^{Lb_2 - 1} + \frac{1}{rV^L}$$

The system (29)-(32) is non-linear in the variables A_1, C_2, V^L and V^H. In order to solve it, the methods illustrated in Dixit (1991) are adapted. Using (31) and (32) to solve for A_1 and C_2 gives (see Appendix):

(33) $$A_1 = \frac{V^{Hb_2} - V^{Lb_2}}{rb_1 (V^{Hb_2} V^{Lb_1} - V^{Hb_1} V^{Lb_2})}$$

(34) $$C_2 = \frac{V^{Hb_1} - V^{Lb_1}}{rb_2(V^{Hb_2}V^{Lb_1} - V^{Hb_1}V^{Lb_2})}$$

Let $K \equiv (V^{Hb_2}V^{Lb_1} - V^{Hb_1}V^{Lb_2})$. Replace A_1 and C_2 into equations (29) and (30) to have:

(35) $$b_2(V^{Hb_2} - V^{Lb_2})V^{Hb_1} = b_1(V^{Hb_1} - V^{Lb_1})V^{Hb_2} + b_1 b_2 K \ln V^H - rb_1 b_2 KI$$

(36) $$b_1(V^{Hb_1} - V^{Lb_1})V^{Lb_2} + b_1 b_2 K \ln V^L = b_2(V^{Hb_2} - V^{Lb_2})V^{Lb_1} - rb_1 b_2 KE$$

Adding equations (35) and (36) and rearranging gives:

(37) $$(V^{Hb_2} - V^{Lb_2})(V^{Hb_1} - V^{Lb_1})(b_2 - b_1) - b_1 b_2 K \ln(V^H / V^L) = -rb_1 b_2 K(I + E)$$

Let:

(38) $$M \equiv (V^H \cdot V^L)^{1/2}$$

(39) $$z \equiv \frac{1}{2} \cdot \ln\left(\frac{V^H}{V^L}\right)$$

Then:

(40) $$e^z = \left(\frac{V^H}{V^L}\right)^{1/2}$$

(41a) $$V^H = Me^z$$

(41b) $$V^L = Me^{-z}$$

Replacing (40), (41a) and (41b) into (37) and simplifying obtains:

(42) $$(b_2 - b_1)(e^{zb_2} - e^{-zb_2})(e^{zb_1} - e^{-zb_1}) - 2b_1 b_2 z(e^{zb_2} e^{-zb_1} - e^{zb_1} e^{-zb_2}) =$$
$$= -rb_1 b_2 (e^{zb_2} e^{-zb_1} - e^{zb_1} e^{-zb_2})(I + E)$$

Use $\sinh(x) = (e^x - e^{-x})/2$ (see e.g. Smirnov, chapter 17) to obtain:

(43) $$2(b_2 - b_1)\cdot \sinh(zb_2)\cdot \sinh(zb_1) - 2b_1 b_2 z \cdot \sinh(z(b_2 - b_1)) =$$
$$= -rb_1 b_2 \cdot \sinh(z(b_2 - b_1))\cdot (I + E)$$

Equation (43) can be evaluated by using a Taylor expansion about the point z=0, noting that $d\sinh(x)/dx = \cosh(x)$ and $d\cosh(x)/dx = \sinh(x)$ where $\cosh(x) = (e^x + e^{-x})/2$ (Smirnov, chapter 17), to obtain (see Appendix):

(44) $$z^3 + \frac{r(I + e)(b_2 - b_1)^2}{4b_1 b_2}\cdot z^2 + \frac{3r(I + E)}{2b_1 b_2} = 0$$

Using Cardano's formula (see Kurosh, chapter 9), the cubic equation (44) has one real root and two complex conjugate roots. The real root of the equation is (see Appendix):

(45) $$z = \sqrt[3]{-\frac{q}{2} + \sqrt{-\frac{D}{108}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{-\frac{D}{108}}} - \frac{r(I + E)(b_2 - b_1)^2}{12 b_1 b_2}$$

where

(46) $$q = \frac{r^3(I+E)^3(b_2-b_1)^6}{864 b_1^3 b_2^3} + \frac{3r(I+E)}{2b_1 b_2}$$

(47) $$D = -\frac{243 r^2 (I+E)^2}{4 b_1^2 b_2^2} - \frac{9}{32}\frac{r^4(I+E)^4(b_2-b_1)^6}{b_1^4 b_2^4}$$

It is possible to show (see Appendix) that z>0. This is an important result as it implies that $V^H > V^L$. Therefore, there exists a range of values for the wage differential in which it is optimal to maintain the status quo, that is, households do not engage in migration in either direction. Households are reluctant to respond to small changes in the wage differential preferring to wait until the wage gap is large enough for migration to be optimal.

A further result is that z is an increasing function of I and of E, the costs of initial and return migration. Hence, an increase in either the cost of migration, or the cost of return migration, or both, increases the range of values in which migration does not take place.

These results are similar to those of insider-outsider theory in labour economics, where increases in hiring and firing costs make the firm more reluctant to hire or fire labour in response to fluctuations in demand for its output (see Lindbeck and Snower, 1988). Firms are reluctant to fire workers in a recession if they perceive the recession to be temporary. Firing workers would increase the costs of the firm, as redundancy obligations in the form of severance pay would need to be met; furthermore, once the recession is over, the firm would incur hiring costs. If firms believe the recession to be long term, workers are fired. In our model of migration and return migration, *I* and *E* respectively, are analogous to the hiring and firing costs of the firm. A small wage differential that is expected to be permanent will generate a large present value. In this case, a household is willing to take part in migration and incur the cost *I* if they are leaving the village of origin, or they incur the cost *E* if they are planning to return. If households observe a large positive wage differential but expect it to be transitory they are unlikely to migrate. Conversely, a large negative differential that is expected to be transitory will not prompt return migration.

3. Migration and risk aversion

An important feature of behaviour under uncertainty is aversion to risk. In this section the model incorporates risk aversion in household behaviour. For analytical simplicity, the option to return migrate is not considered. The approach to risk aversion presented in this section is very general, and could easily be adapted to analyse different forms of uncertainty.

Migration is by its very nature risky. Once a household member migrates, there is a decline in current household income. The greater the marginal product of the migrant, the larger is this decline. It should be noted that a household member with a large marginal product in the village of origin would not necessarily have a large marginal product in the village of destination. There is always a risk that the migrant cannot substantially augment family income. Income in the village of origin can be uncertain (for instance, a bad harvest), but equally there is no certainty on income in the destination village.

A risk-neutral household is indifferent to fluctuations in income, if the expected value of income is unchanged. By contrast, a risk-averse household prefers a steady stream of income rather than a fluctuating income flow, even if the expected *NPV* were to be the same. For a risk-averse household, a steady stream of income yields a higher lifetime utility, and so migration can be a strategy to smooth income fluctuation.

A risk-averse household will be more cautious about moving given the irreversibility of the migration decision. The sunk costs involved cannot be recovered at a future date and therefore there is a higher opportunity cost attached to migrating now. Hence, households might display a greater degree of inertial behaviour.

Under risk aversion, the instantaneous household utility can be modelled as:

(48) $$U = U(W)$$

where U is increasing and concave, such that $U' > 0$, $U'' < 0$.

As before, the exponential of the wage differential, $V = e^{W^D - W^O}$, follows a geometric Brownian motion:

(49) $$dV = s_v V dz_v$$

where the disturbance dz_v follows a white-noise stochastic Wiener process:

(50) $$dz_v(t) = e_v(t) \cdot \sqrt{dt}$$

where

(51) $$e_v(t) \sim N(0,1)$$

is a serially uncorrelated stochastic process. Equations (50) and (51) imply:

(52) $$dz_v \sim N(0, dt)$$

Let I be the migration cost. The value of the migration opportunity is:

(53) $$F = F(e^{W^D - W^O}) = F(V)$$

The Bellman equation is:

(54) $$rF(V)dt = U(W^O)dt + E[dF(V)]$$

Using Itô's Lemma, taking expectations, replacing into the Bellman equation and rearranging the following second-order differential equation in the value function $F(V)$ is obtained :

(55) $$\frac{1}{2}s_v^2V^2F_{vv} - rF = -U(W^O)$$

Proceeding as in section 2, the general solution for the inhomogeneous equation (57) is:

(56) $$F(V) = A_1V^{b_1} + A_2V^{b_2} + \frac{U(W^O)}{r} \qquad V \in (0, V^*)$$

where $b_1 > 1$ and $b_2 < 0$ are defined in section 2, equations (14a) and (14b) respectively, and where *V** is the critical threshold of the wage differential.

The optimal migration strategy must have the form: do not migrate if $V \in (0, V^*)$, migrate if $V \in [V^*, \infty)$. Consider the constant A_2: as $V \to 0$, W^D - W^O $\to -\infty$ and therefore the option to migrate should be worthless. Since $b_2 < 0$, in order to avoid $A_2V^{b_2} \to \infty$ as $V \to 0$ A_2 must be set to equal zero i.e. $A_2 = 0$. Hence the general solution to the differential equation (56) is:

(57) $$F(V) = A_1V^{b_1} + \frac{U(W^O)}{r} \qquad V \in (0, V^*)$$

The values of the coefficient A_1 and of the critical threshold V^* are obtained from the value-matching and the smooth-pasting condition. Under risk aversion, the value-matching condition is:

(58) $$F(V^*) = \frac{U(W^D)}{r} - I$$

and the smooth-pasting condition is:

(59) $$F_v(V^*) = \frac{U'(W^D)}{r}$$

Using (57) and the definition $V = e^{W^D - W^O}$ the following system is obtained:

(60) $$A_1 V^{*b_1} + \frac{U(W^O)}{r} = \frac{U(\ln V^* + W^O)}{r} - I$$

(61) $$A_1 b_1 V^{*(b_1 - 1)} = \frac{U'(\ln V^* + W^O)}{rV^*}$$

Substituting out $A_1 V^{*b_1}$ from (60) and (61) and equating the following implicit function in V^* can be written:

(62) $$f(V^*) = U'(\ln V^* + W^O) - b_1 \left[U(\ln V^* + W^O) - U(W^O)\right] + rb_1 I = 0$$

By the implicit function theorem, the following is obtained:

(63) $$\frac{dV^*}{dI} = -\frac{\partial f / \partial I}{\partial f / \partial V^*} = -\frac{rb_1}{\left[U''(W^D) - b_1 U'(W^D)\right] / V^*} > 0$$

Equation (63) shows that an increase in the cost of migration, I, will increase the critical value V^* and thus delay the decision to migrate. This is because the region where it is optimal not to migrate has widened. Similarly,

(64) $$\frac{dV^*}{dr} = -\frac{\partial \phi / \partial r}{\partial \phi / \partial V^*} = -\frac{\beta_1 I + rI \cdot \partial \beta_1 / \partial r}{\left[U''(W^D) - \beta_1 U'(W^D)\right]/V^*} > 0$$

Equation (64) shows that an increase in the rate of interest, r, also has the effect of delaying migration. This finding is consistent with intuition, in that one would expect high interest rates to act as a deterrent in the migration decision.

To evaluate the response of V^* to changes in the variance of the instantaneous shocks, σ_v^2, note that:

(65) $$\frac{\partial \phi}{\partial \sigma_v^2} = \frac{\partial \phi}{\partial \beta_1} \cdot \frac{\partial \beta_1}{\partial \sigma_v^2}$$

From equation (14a) of section 2, $\partial \beta_1 / \partial \sigma_v^2 < 0$. The partial derivative $\partial \phi / \partial \beta_1$ gives:

(66) $$\frac{\partial \phi}{\partial \beta_1} = -\left[U(W^D) - U(W^O)\right] + rI < 0 \quad \Leftrightarrow \quad U(W^D) - U(W^O) > rI$$

and therefore:

$$(67) \qquad \frac{dV^*}{ds_v^2} = -\frac{\P f / \P s_v^2}{\P f / \P V^*} > 0 \qquad \Leftrightarrow \qquad U(W^D) - U(W^O) > rI$$

The intuition for this result is as follows. Suppose W^O > W^D: in the absence of stochastic shocks, it would never be profitable to migrate since $U(W^D) - U(W^O) < rI$. With positive shocks, as the variance s_v^2 increases there is an increased probability that the wage of destination W^D will climb above the wage of origin W^O, and therefore migration would be more attractive. This would result in a decline of the critical value V^* (the set of values of V for which migration is not optimal will be smaller). Conversely, when $U(W^D) - U(W^O) > rI$ an increase in the variance of the stochastic shocks will make it more likely for the destination wage to fall below the wage of origin, thereby discouraging migration.

Consider now the effect of an increase in the degree of risk aversion on the decision to migrate. The coefficient of relative risk aversion is defined as (see e.g. Laffont, 1991, page 24):

$$(68) \qquad g(W) = -\frac{W \cdot U''(W)}{U'(W)}$$

One has:

$$(69) \qquad \frac{\P f}{\P V^*} = \frac{U'(W)}{V^*}\left[\frac{W \cdot U''(W)}{U'(W)} \cdot \frac{1}{W} - b_1\right]$$

$$= -\frac{U'(W)}{V^*}\left[\frac{g(W)}{W} + b_1\right] < 0$$

For a risk-neutral household, $\gamma = 0$. For a risk-averse household, $\gamma > 0$. From (69), the absolute value $\left|\partial \phi / \partial V^*\right|$ is thus an increasing function of the degree of risk aversion. Hence, by the implicit function theorem the effect on V^* of changes in the parameters I, r and σ_v^2 is magnified by risk aversion.

The importance of this result is twofold. Firstly, it allows us to establish the role of risk aversion in the decision-making process. For instance, section 2 showed that an increase in migration cost I makes the household more reluctant to migrate, by raising the critical threshold V^*. In the presence of risk aversion, the critical value V^* is raised even further by increases in I. Risk aversion therefore exacerbates the effects of those parameters that affect migration. In other words, the qualitative effects are unchanged, but the quantitative effects are stronger.

Secondly, the degree of risk aversion can be an important source of heterogeneity across households. This means that, given a positive wage differential, a household that is more risk averse will be more cautious about migrating than a household that is less risk averse, even if the cost of migration or any of the other parameters influencing migration is the same for both households.

4. Rural-rural migration and uncertainty on the wage differential

The stability of agricultural income in the face of shocks can be an important determinant of rural-rural migration. Technology may be a means of trying to ensure stability. Government investment, used either to implement new technology (for example, the Green Revolution in India), or for the development of non-agricultural activities, can result in structural changes in the rural economy. Given that rural-urban flows have reached critical levels in many cities in developing countries and that the explanation for these flows is the inter-sectoral wage differential, then investment in the rural area can be seen as a strategy to reduce the wage differentials and so stem the rural-urban flow. However, the effect of government investment can not only augment wages in a rural area, but it may also reduce the uncertainty of this wage thereby prompting rural-rural migration flows.

Two rural areas may yield equivalent net present values of expected future incomes, but one of the areas may be characterised by a higher degree of income uncertainty than the other. In this case, one would expect the migrant to move to the area that offers the more stable income prospects.

This issue is analysed by considering a *neutral spread* of the stochastic process of the destination wage [2]. Consider an initial stochastic process and transform it by adding uncorrelated random noise, which has the effect of increasing the variability of the destination wage. The initial destination wage and the new destination wage will both yield the same net present value. However, a rational migrant will prefer the destination that gives greater security in terms of future expected incomes. It is shown that the addition of a neutral spread to the destination wage process raises the critical threshold value of the wage differential at which it is optimal to migrate. Increased uncertainty in the destination wage has the effect of delaying the time at

[2] The notion of neutral spread is due to Ingersoll and Ross (1992).

which it is optimal to migrate. Our analysis is conducted for a risk-neutral migrant, but the results from section 3 suggest that risk aversion would exacerbate the effects of a neutral spread.

Consider an increase in the uncertainty associated with the wage in the destination area. The increase in uncertainty is modelled as a neutral spread of the stochastic process describing the destination wage, W^D. A neutral spread can be regarded as the dynamic extension to stochastic processes of the mean-preserving spread for static random variables (the latter concept is due to Rothschild and Stiglitz, 1970). The stochastic process describing the destination wage $\{W^D(t)\}_{t=0}^{\infty}$ is augmented by an uncorrelated white noise stochastic process, $\{h(t)\}_{t=0}^{\infty}$. The destination wage after the neutral spread becomes:

(70) $$W^{hD}(t) = W^D(t) + h(t)$$

where the neutral spread is such that its expectation is equal to zero and its increments are uncorrelated with the increments of the process W^D:

(71) $$E[h] = 0, \qquad E[dW^D, dh] = 0$$

In order to assess the impact of the neutral spread on the decision to migrate, the value to the household of having the migration opportunity before and after the neutral spread is computed. It is shown that a neutral spread increases the value to the household of keeping open the option to migrate in the future.

As in section 2, the value to the household of having the migration opportunity is defined as:

(72) $$F = F(e^{W^D - W^O}) = F(V)$$

The value-matching condition is:

(73) $$F(V^*) = E[\text{PV}(W^D)] - I$$

Let $F_0(V)$ be the value of the migration opportunity at $t=0$ before the spread:

(74) $$F_0(V^*) = E[\text{PV}_0(W^D)] - I$$

Let $F_0^h(V)$ be the value of the migration opportunity at $t=0$ after the spread:

(75) $$F_0^h(V^*) = E[\text{PV}_0(W^{hD})] - I = E[\text{PV}_0(W^D)] + E[\text{PV}_0(h)] - I$$
$$= E[\text{PV}_0(W^D)] - I$$

by (71).

Let F_t be the value at time $t = 0$ of the opportunity to migrate at time $t \geq 0$ before the spread:

(76) $$F_t = E\left\{ e^{-\int_0^t r(s)ds} \left[\mathrm{PV}_t (W^D) - I \right] \right\}$$

and let F be the value at time *t = 0* of the opportunity to migrate at *any* time *t ³ 0* before the spread:

(77) $$F = \sup_{t \geq 0} F_t$$

Let now F_t^h be the value at time *t = 0* of the opportunity to migrate at time *t ³ 0* after the spread:

(78) $$F_t^h = E\left\{ e^{-\int_0^t r(s)ds} \left[\mathrm{PV}_t (W^{hD}) - I \right] \right\}$$

and let F^h be the value at time $t = 0$ of the opportunity to migrate at *any* time $t \geq 0$ after the spread:

(79) $$F^h = \sup_{t \geq 0} F_t^h$$

The following definition is made: $t^* = \arg\max(F_t)$, that is, $F_{t^*} \geq F_t$, $\forall t \geq 0$. It follows that:

$$F^h \geq F_{t^*}^h$$

$$= E\left\{ e^{-\int_0^{t^*} r(s)ds} \left[\mathrm{PV}_{t^*} (W^{hD}) - I \right] \right\}$$

$$= E\left\{ e^{-\int_0^{t^*} r(s)ds} \left[\text{PV}_{t^*}(W^D) + \text{PV}_{t^*}(h) - I \right] \right\}$$

$$= E\left\{ e^{-\int_0^{t^*} r(s)ds} \left[\text{PV}_{t^*}(W^D) - I \right] \right\} + e^{-\int_0^{t^*} r(s)ds} E\{\text{PV}_{t^*}(h)\}$$

$$= E\left\{ e^{-\int_0^{t^*} r(s)ds} \left[\text{PV}_{t^*}(W^D) - I \right] \right\}$$

(80) $= F$

where the second line of (80) follows from equation (78), the third line from the definition (70), the fourth line from the additivity property of the expectation operator, the fifth line from the first of conditions (71), and finally the last line follows from the definitions (76) and (77).

Equation (80) shows that a neutral spread increases the value to the household of keeping open the option to migrate in the future. It should be noted that this result would be enhanced under risk aversion. A positive net present value is insufficient to motivate migration. Crucially, households consider the extent of uncertainty associated with income.

5. Conclusions

This paper considers migration as an investment decision. A continuous-time stochastic model is used to explain the optimal timing of migration, in the presence of ongoing uncertainty over wage differentials. Out-migration and return migration are jointly explored. How the option to return to the village of origin may affect the initial decision to migrate is examined. The effect of risk aversion on the propensity to migrate is analysed.

The results obtained show that households will prefer to wait before migrating even in the presence of a positive wage differential because of the uncertainty and the sunk costs associated with migration. Similarly, households in the destination area will prefer to wait before returning to the village of origin even if the wage differential becomes negative. Hence, there is a region of inertia where households do not migrate in either direction. The optimal location of the household is dependent on past household migration decisions, *i.e.*, there is a hysteresis effect (see Dixit, 1992). The degree of inertia is shown to be an increasing function of costs of migration.

Although migration has been observed to take place in the presence of small wage differentials, this should not be taken to imply that wage differentials are not important. Households may be forced to migrate because of distress factors (in the context of this model this means that F^O is reduced and, hence, V^H is lower) or they prefer a small wage differential that is persistent to large wage differentials that are only temporary.

The option to migrate is delayed under risk aversion. The critical threshold of the wage differential for which it is optimal to migrate is raised. There is an increased value in waiting than under risk neutrality. An increased degree of risk aversion discourages migration, and interacts with the other variables and parameters affecting migration by exacerbating their effects.

A neutral spread of the destination wage is considered. Increased uncertainty discourages migration into rural areas with a less predictable income profile. This result can explain why some rural areas attract a higher number of migrants than others.

This approach has allowed a rigorous examination of the effects of uncertainty and risk aversion on the household decision-making process. In addition, features of rural-rural migration can be analysed, which have hitherto been ignored in the literature. The application of a neutral spread can be considered both in the rural-rural context and in the rural-urban context [3]. The generality of this model means that migration can be studied in different contexts.

[3] See Khwaja (2000b).

Appendix

Section 2.
Solution of equations (31) and (32), yielding solutions for A_1 and C_2.

The determinant of the system is:

(A1) $$\Delta = b_1 b_2 (V^{Hb_2-1} V^{Lb_1-1} - V^{Hb_1-1} V^{Lb_2-1}$$

Using Cramer's rule,

(A2) $$A_1 = \frac{-\frac{1}{rV^H} b_2 V^{Lb_2-1} + \frac{1}{rV^L} b_2 V^{Hb_2-1}}{\Delta}$$

$$= \frac{V^{Hb_2} - V^{Lb_2}}{rb_1 (V^{Hb_2} V^{Lb_1} - V^{Hb_1} V^{Lb_2})}$$

(A3) $$C_2 = \frac{\frac{1}{rV^L} b_1 V^{Hb_1-1} - \frac{1}{rV^H} b_1 V^{Lb_1-1}}{\Delta}$$

$$= \frac{V^{Hb_1} - V^{Lb_1}}{rb_2 (V^{Hb_2} V^{Lb_1} - V^{Hb_1} V^{Lb_2})}$$

Taylor expansion of equation (43).

Write equation (43) as

(A4) $$L(z) = R(z)$$

where

(A5) $$L(z) = L_1(z) + L_2(z)$$

(A6) $$L_1(z) = 2(b_2 - b_1) \cdot \sinh(zb_2) \cdot \sinh(zb_1)$$

(A7) $$L_2(z) = -2b_1 b_2 z \cdot \sinh(z(b_2 - b_1))$$

(A8) $$R(z) = -rb_1 b_2 \cdot \sinh(z(b_2 - b_1)) \cdot (I + E)$$

Compute Taylor's expansion about the point z=0:

(A9) $$L_1(z) = L_1(0) + L_1'(0)z + L_1''(0)\frac{z^2}{2} + L_1'''(0)\frac{z^3}{3!} + L_1''''(0)\frac{z^4}{4!} + ...$$

and similarly for $L_2(z)$, $R(z)$. Note that

(A10) $$L_1(0) = 0$$

(A11) $$L_1'(z) = 2(b_2 - b_1)[b_2 \cosh(zb_2)\sinh(zb_1) + b_1 \sinh(zb_2)\cosh(zb_1)]$$

(A12) $$L_1'(0) = 0$$

(A13) $$L_1''(z) = 2(b_2 - b_1)[b_2^2 \sinh(zb_2)\sinh(zb_1) + b_1 b_2 \cosh(zb_2)\cosh(zb_1)$$
$$+ b_1 b_2 \cosh(zb_2)\cosh(zb_1) + b_1^2 \sinh(zb_2)\sinh(zb_1)]$$
$$= 2(b_2 - b_1)[b_2^2 \sinh(zb_2)\sinh(zb_1) + 2b_1 b_2 \cosh(zb_2)\cosh(zb_1)$$
$$+ b_1^2 \sinh(zb_2)\sinh(zb_1)]$$

(A14) $$L_1''(0) = 4b_1 b_2 (b_2 - b_1)$$

(A15) $$L_1'''(z) = 2(b_2 - b_1)[b_2^3 \cosh(zb_2)\sinh(zb_1) + 3b_1 b_2^2 \sinh(zb_2)\cosh(zb_1)$$
$$+ 3b_1^2 b_2 \cosh(zb_2)\sinh(zb_1) + b_1^3 \sinh(zb_2)\cosh(zb_1)]$$

(A16) $$L_1'''(0) = 0$$

(A17) $$L_1''''(z) = 2(b_2 - b_1)[b_2^4 \sinh(zb_2)\sinh(zb_1) + 4b_1 b_2^3 \cosh(zb_2)\cosh(zb_1)$$
$$+ 6b_1^2 b_2^2 \sinh(zb_2)\sinh(zb_1) + 4b_1^3 b_2 \cosh(zb_2)\cosh(zb_1)$$
$$+ b_1^4 \sinh(zb_2)\sinh(zb_1)]$$

(A18) $$L_1''''(0) = 8(b_2 - b_1)b_1 b_2 (b_1^2 + b_2^2)$$

(A19) $$L_2(0) = 0$$

(A20) $$L_2'(z) = -2b_1 b_2[\sinh(z(b_2 - b_1)) + z \cdot \cosh(z(b_2 - b_1)) \cdot (b_2 - b_1)]$$

(A21) $$L_2'(0) = 0$$

(A22) $$L_2''(z) = -2b_1 b_2[\cosh(z(b_2 - b_1)) \cdot (b_2 - b_1) + \cosh(z(b_2 - b_1)) \cdot (b_2 - b_1)$$
$$+ z \cdot \sinh(z(b_2 - b_1)) \cdot (b_2 - b_1)^2]$$

(A23) $$L_2''(0) = -4b_1 b_2 (b_2 - b_1)$$

(A24)
$$L_2'''(z) = -2b_1 b_2 (b_2 - b_1)[2\sinh(z(b_2 - b_1)) \cdot (b_2 - b_1) + \sinh(z(b_2 - b_1)) \cdot (b_2 - b_1)$$
$$+ z \cdot \cosh(z(b_2 - b_1)) \cdot (b_2 - b_1)^2]$$

(A25) $$L_2'''(0) = 0$$

(A26) $$L_2''''(z) = -2b_1 b_2 (b_2 - b_1)^2 [3\cosh(z(b_2 - b_1)) \cdot (b_2 - b_1)$$
$$+ \cosh(z(b_2 - b_1)) \cdot (b_2 - b_1) + \sinh(z(b_2 - b_1)) \cdot (b_2 - b_1)^2]$$

(A27) $$L_2''''(0) = -8b_1 b_2 (b_2 - b_1)^3$$

(A28) $$R(0) = 0$$

(A29) $$R'(z) = -rb_1 b_2 (I + E)(b_2 - b_1) \cdot \cosh(z(b_2 - b_1))$$

(A30) $$R'(0) = -rb_1 b_2 (I + E)(b_2 - b_1)$$

(A31) $$R''(z) = -rb_1 b_2 (I + E)(b_2 - b_1)^2 \cdot \sinh(z(b_2 - b_1))$$

(A32) $$R''(0) = 0$$

(A33) $$R'''(z) = -rb_1 b_2 (I + E)(b_2 - b_1)^3 \cdot \cosh(z(b_2 - b_1))$$

(A34) $$R'''(0) = -rb_1 b_2 (I + E)(b_2 - b_1)^3$$

(A35) $$R''''(z) = -rb_1 b_2 (I + E)(b_2 - b_1)^4 \cdot \sinh(z(b_2 - b_1))$$

(A36) $$R''''(0) = 0$$

The Taylor expansion up to the 4th order takes the form:

(A37) $$\frac{1}{3}(b_2 - b_1) b_1 b_2 (b_1^2 + b_2^2) z^4 - \frac{1}{3}(b_2 - b_1)^3 b_1 b_2 z^4 =$$
$$= -rb_1 b_2 (b_2 - b_1)(I + E)\left[1 + \frac{z^2}{6}(b_2 - b_1)^2\right] z$$

The solution z =0 would not be acceptable, since it would contradict (29) and (30) (unless I=E=0). Divide (A37) by $b_1 b_2 (b_2 - b_1) z$:

(A38) $$\frac{1}{3}(b_1^2 + b_2^2)z^3 - \frac{1}{3}(b_2 - b_1)^2 z^3 = -r(I+E)\left[1 + \frac{z^2}{6}(b_2 - b_1)^2\right]$$

Rearrange to obtain:

(A39) $$z^3 + \frac{r(I+E)(b_2 - b_1)^2}{4b_1 b_2} z^2 + \frac{3r(I+E)}{2b_1 b_2} = 0$$

Let

(A40) $$y = z + \frac{r(I+E)(b_2 - b_1)^2}{12 b_1 b_2}$$

Then the cubic equation becomes:

(A41) $$y^3 + py + q = 0$$

where

(A42) $$p = -\frac{r^2 (I+E)^2 (b_2 - b_1)^4}{48 b_1^2 b_2^2}$$

(A43) $$q = \frac{r^3 (I+E)^3 (b_2 - b_1)^6}{864 b_1^3 b_2^3} + \frac{3r(I+E)}{2b_1 b_2}$$

The discriminant of (A41) is defined as (Kurosh, 1977, chapter 9):

(A44) $$D = 4p^3 - 27q^2$$

$$= -\frac{243 r^2 (I+E)^2}{4 b_1^2 b_2^2} - \frac{9}{32}\frac{r^4 (I+E)^4 (b_2 - b_1)^6}{b_1^4 b_2^4}$$

$$< 0$$

Since $D<0$, the cubic equation has one real root and two complex conjugate roots. Let

(A45) $$a = \sqrt[3]{-\frac{q}{2} + \sqrt{-\frac{D}{108}}}$$

(A46) $$g = \sqrt[3]{-\frac{q}{2} - \sqrt{-\frac{D}{108}}}$$

The real root for y is:

(A47) $$y = a + g$$

The real root for *z* can therefore be written as:

(A48) $$z = \sqrt[3]{h_1^3 + h_2 + h_3} + \sqrt[3]{h_1^3 + h_2 - h_3} + h_1 > 0$$

where

(A49) $$h_1 = -\frac{r(I + E)(b_2 - b_1)^2}{12 b_1 b_2} > 0$$

(A50) $$h_2 = -\frac{3r b_1^2 b_2^2 (I + E)}{4 b_1^3 b_2^3} > 0$$

(A51) $$h_3 = \frac{r(I + E)}{96 b_1^2 b_2^2}\sqrt{648 + 3r^2 (I + E)^2 (b_2 - b_1)^6} > 0$$

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