Social Time Preference

by

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Abstract

The observed practice of discounting the future should not be rationalised on the grounds of myopia or selfishness. A positive rate of pure time preference is necessary to ensure that heterogeneous generations are treated in an egalitarian fashion. A zero social discount rate would yield intertemporal allocations which are biased against the current generations. Endogenous productivity growth requires that the social discount rate be set above the subjective rate of pure time preference. Positive social time preference, far from discriminating against future generations, is essential for a fairer intertemporal allocation of resources.

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1. Introduction

A positive rate of time preference has often been rationalised on the basis of either myopic behaviour or deliberate selfishness of current generations. A popular argument is that future generations ought to be given exactly the same weight as the currently alive ones: in other words, there should be no discounting of future relative to present utility. The debate on the ethical motives prescribing a zero discounting of future utilities dates back at least to the seminal work by Pigou (1920) and Ramsey (1928). Positive discounting has, on the other hand, been proved to be necessary for a well-defined representation of preferences over an infinite horizon (see Koopmans, 1960; Diamond, 1965). A sufficiently high social discount rate is also essential to prevent present generations from being unfairly treated. As argued by Mirrlees (1967), the generations currently alive could have an exceedingly low consumption per capita under zero discounting (see also Chakravarty, 1969). A positive discount rate could thus act as a compensating device against the built-in bias in favour of future generations, which occurs when technology changes over time. Dasgupta and Heal (1979, chapter 9) also argue that, in order to assess the validity of discounting future utilities, it is necessary to consider the implications of alternative social discount rates about future growth paths.

In the present paper, we find the optimal social discount rate by following the approach originally suggested by Calvo and Obstfeld (1988) where utilities of different generations are discounted back to their birth date. We are able logically to distinguish between the social and the private discount rates. With exogenous technical progress, the optimal value for the social discount rate is the instantaneous rate of productivity growth, augmented by the rate of growth of population. With endogenous productivity growth, the social discount rate must be equal to the marginal social product of capital.

The scheme of the paper is as follows. Section 2 briefly reviews the issue of discounting the future and sets forth our model. The time-consistent utilitarian criterion is described in detail and the optimal value of the discount rate is explicitly derived under exogenous productivity growth. Section 3 extends the analysis to the case of endogenous productivity growth. Section 4 sums up the main results.

2. Overlapping generations and discounting

Optimum growth theory has largely concentrated on the maximisation of a utility function over an infinite time horizon, where a benevolent social planner typically discounts future utilities at a positive rate. Pigou (1920, p. 29) and Ramsey (1928) are strongly opposed to discounting, on the grounds that it is not ethical to attach a lower weight to the welfare of future generations.

In the literature there are however also arguments in favour of discounting future utilities[1]. For example, when the discount rate is set equal to zero in the objective function of the social planner, the resulting inter-temporal consumption allocation is biased against the current generations and excessively favours the future ones (see Mirrlees, 1967, p. 112; Chakravarty, 1969, section 3.4). The use of a positive discount rate in the social objective function is also consistent with Koopmans' (1960) preference ordering over the set of well-being paths.

The literature has mainly concentrated on the representative agent framework, thus neglecting the heterogeneity of agents. Heterogeneity across generations at each moment in time can be modelled by employing the continuous-time model originally developed by Yaari (1965) and Blanchard (1985), and later extended by Weil (1989) and Buiter (1988) to allow for a more flexible demographic structure.

The instantaneous population growth rate is denoted by $n=b-l$, where b and l are, respectively, instantaneous birth and death rates. The private subjective rate of pure time preference is $r > 0$ [2]. The individual consumer born at time $s \leq t$ maximises lifetime utility

$$\max_{\{\bar{c}(s,v)\}} \int_t^\infty u(\bar{c}(s,v)) e^{-(r+l)(v-t)} dv \tag{1}$$

where $u(\cdot)$ in the instantaneous felicity function ($u'(\cdot)>0$, $u''(\cdot)<0$), subject to the intertemporal budget constraint:

$$\int_t^\infty \bar{c}(s,v) e^{-\int_t^v [r(y)+l]dy} dv \leq \bar{h}(s,t) + \bar{a}(s,t) \tag{2}$$

where $\bar{h}(s,t)$ is human and $\bar{a}(s,t)$ is non-human wealth.

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[1] Ramsey himself (1931, p. 291) has acknowledged that it is necessary to apply some perspective to time: see the citation and comments in Arrow and Kurz (1970, p. 12).

[2] The case for a positive private intertemporal discount rate appears to be compelling, as persuasively argued by Olson and Bailey (1981).

The Euler equation for optimal consumption over time yields the Keynes-Ramsey condition:

$$-\frac{u_{cc}\dot{\bar{c}}}{u_c} = (r - r) \qquad (3)$$

An aggregate variable $X(t)$ in the model is defined as:

$$X(t) = be^{-lt} \int_{-\infty}^{t} \bar{x}(s,t)e^{bs}ds \qquad (4)$$

Following Solow (1956), technology is described by a constant-returns-to-scale aggregate production function:

$$Y(t) = F[K(t), e^{pt}L(t)] \qquad (5)$$

where $Y(t)$ is aggregate output, $F(\cdot,\cdot)$ is the production function satisfying Inada's conditions, $K(t)$ and $L(t)$ are capital stock and labour force (assumed equal to population), and p is the instantaneous rate of labour-augmenting productivity growth.

An appropriate social welfare function must meet the requirements of time consistency and of symmetry across generations. This requires each generation's utility to be discounted back to its birth date, rather than to the current date (Calvo and Obstfeld, 1988). The optimality criterion takes the form:

$$\Omega(t) = \int_{t}^{\infty} \left\{ \int_{0}^{\infty} u[\bar{c}(v-h,v)]e^{[(d-n)-(r+l)]h}dh \right\} e^{-(d-n)v}dv \qquad (6)$$

where $\Omega(t)$ is the social objective function and $h = v - s$ is individual age.

The intertemporal allocation problem for the social planner consists of maximising the function

$$\Omega(t) = \int_{t}^{\infty} U[c(v)]e^{-(d-n)v}dv \qquad (7)$$

where the utility functional $U[c(v)]$ solves the static cross-sectional allocation problem across all generations alive at time v:

(8) $$U[c(v)] = \max_{\{\bar{c}(v-h,v)\}_{h=0}^{\infty}} \int_0^{\infty} u[\bar{c}(v-h,v)] e^{[(d-n)-(r+l)]h} dh$$

The first-order condition for the optimal consumption allocation problem at time t is:

(9) $$u_c[\bar{c}(t-h,t)] e^{(d-n-r)h} = f(t)$$

where $f(t)$ is equal to the marginal utility of average consumption:

(10) $$f(t) = U'[c(t)]$$

The marginal utility of the average consumption index evolves over time according to

(11) $$\frac{\dot{f}}{f} = -\frac{1}{s(c)} \frac{\dot{c}}{c}$$

where $s(c) \equiv -U'(c)/U''(c)c$ is the instantaneous elasticity of substitution of average utility. The condition for the optimal dynamic allocation of consumption is

(12) $$\frac{\dot{c}}{c} = s(c) \cdot (F_K - d)$$

where F_K is the marginal product of capital.

Maximisation of the optimality functional requires the social discount rate to be equalised to the marginal product of capital. Under the standard Inada conditions on $F(\cdot,\cdot)$, positive discounting is therefore necessary according to a utilitarian criterion. A positive d does not reflect either myopic behaviour or an impatience bias at the expenses of future generations.

The optimal level of d can be chosen to maximise average steady-state consumption per capita, c. The golden rule criterion (Phelps, 1961) with exogenous productivity growth requires $F_K = p + n$, whence maximisation of long-run consumption per capita is achieved by setting

(13) $$d = p + n$$

The social rate of pure time preference must be equal to the instantaneous rate of productivity growth, augmented by the rate of population growth.

The intuition for this result is as follows. Under constant productivity growth, aggregate output grows at the rate $p + n$. Thus, $p + n$ measures the social opportunity cost of current relative to future consumption. The allocation of resources consistent with steady-state consumption maximisation requires that the marginal product of capital be equalised to $p + n$, which is achieved on the optimal time-consistent path when $d = p + n$, that is, when the social discount rate is equal to the opportunity cost of current consumption.

The above result shows that a positive social discount rate is necessary in order to maintain constant consumption over time. A zero social discount rate implies an asymmetry across generations, with future generations enjoying higher average consumption levels than the current ones. Thus, far from leading to a fairer allocation of resources over time, the absence of a positive discount rate would favour future generations, at the expenses of the current ones.

3 Endogenous growth and the social discount rate

The analysis can be extended without difficulty to the case of endogenous growth. Consider a prototypical learning-by-doing model, of the variety put forward by Frankel (1962) and Sheshinski (1967) and analysed by Buiter (1993) and by Barro and Sala-i-Martin (1995, section 4.3.5). Firm output is an increasing function of the average capital-labour ratio in the economy:

(14) $$Y_{it} = F(K_{it}, E_{it})$$

where the production function $F(\cdot,\cdot)$ exhibits constant returns to scale, and where E_{it} denotes labour in efficiency units.

Let $y_{it} \equiv Y_{it} / L_{it}$ and $k_{it} \equiv K_{it} / L_{it}$ denote variables per capita, and let $k_t \equiv (1/N)\Sigma k_{it}$ be the average capital-labour ratio in the economy. In a symmetrical equilibrium $k_{it} = k_t$, whence the production function (14) can be written in per capita form as

(15) $$y_{it} = F(k_{it}, k_t) = a k_t$$

where $a \equiv F(1,1)$ measures the social marginal product of capital. For the model to have a non-trivial solution it must be that $a > r$: the private subjective intertemporal discount rate needs to be less than the social marginal product of capital.

The Euler equation for the optimal intertemporal allocation of resources is now

(16) $$\frac{\dot{c}}{c} = s(c) \cdot (a - d)$$

where c is average consumption per capita. Symmetry across generations now requires

(17) $$d = a$$

Therefore, the social discount rate must exceed the private rate of pure time preference in order for an equitable intertemporal allocation of resources to be attained.

In other words, a higher social discount rate is now needed to maintain constant consumption over time. This result provides additional theoretical support to Mirrlees's (1967) intuition that positive discounting is indeed necessary in order to treat equitably the current generation relative to the future ones. An egalitarian allocation of resources over time under endogenous productivity growth requires that a lower amount be saved and invested along the optimal growth path, and this in turn must imply a higher social discount rate.

4. Conclusions

A social rate of pure time preference is justifiable on purely ethical grounds. Intergenerational fairness and time consistency require future utilities to be positively discounted. Thus, the observed practice of discounting the future should not be regarded as evidence of egotistical behaviour by the current generations at the expenses of the well-being of those as yet unborn.

Under exogenous productivity growth, the optimal social discount rate is equal to the sum of the instantaneous rates of productivity and population growth. Under endogenous productivity growth, the optimal social discount rate must be equal to the marginal social product of capital.

Positive social time preference, far from discriminating against future generations, is essential for an equitable intertemporal allocation of resources.

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