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A Note on Credit Allocation, Income Distribution, and the Circuit of Capital

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Paulo L dos Santos(s)*

Abstract

This note considers the relationship between credit allocation and the aggregate, class distribution of income in the Circuit of Capital. Production and consumption credit inject means of purchase in different locations in the monetary circuits of capitalist reproduction. On the basis of comparative-dynamic analysis of the properties of steady-state evolutions, the note shows that production credit increases the wage share in total income, while consumption credit increases profit shares. These findings hold in general for any evolution in which sectoral revenue-elasticities of outlays measure less than unity. The note also motivates a new endogenous approach to the aggregate distribution of income, and offers new analytical tools facilitating work based on demand-constrained models of the Circuit of Capital.

Keywords: Credit and Income Distribution, Circuit of Capital , Marxian Analyses

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This note originated in discussions on the Circuit of Capital, credit allocation, and income distribution with Deepankar Basu. In many ways it is an answer to some of the questions thrown up by that conversation. I am also indebted to Duncan Foley, for his framework and his patience in our ongoing discussions on this and related issues. Peter Skott and Anwar Shaikh have also provided useful comments on this work. Any errors below are my own.

1. Introduction

This note explicitly considers the structural impact of credit allocation on the class distribution of income. It does so in response to an apparent analytical gap in extant discussions of income distribution and household credit. Rising levels of income inequality have been understood as *causes* of recent increases in levels of household credit by contributions spanning a broad range of analytical traditions. Based on a dynamic stochastic general equilibrium model, Iacoviello (2008) argues and empirically supports the contention that, faced with widening income distributions, poorer households have sought to reduce widening consumption gaps through increased levels of net borrowing. From a Keynesian-underconsumptionist perspective, Barba and Pivetti (2008) locate in this type of borrowing a problematic and unsustainable "solution" to the demand shortfalls seen to arise from falling wages in the US economy. The same broad point was motivated earlier, from a Marxian-underconsumptionist perspective, by Bellamy Foster (2006).

Inasmuch as household indebtedness and the asset holdings that support lending to households are clustered towards opposite sides of the income distribution, interest-payment flows posed by household debt increase inequality. A range of critical and heterodox contributions have considered the significance of the resulting redistributions. Dutt (2006), for instance, argues that by transferring income to households with lower consumption propensities, this lending may eventually sap demand and prospects for growth. From a different analytical perspective, Lapavitsas (2009) and dos Santos (2009) documented the significance of the transfers of value effected by this lending for households and for banking organisations, and interpreted them as an important, financial form of expropriation characteristic of contemporary capitalism.[2] Pressman and Scott (2009) notably advocated the inclusion of net interest payments in estimations of income distribution in the US. But the possibility that credit extended to households may help shape the aggregate distribution of income through mechanisms other than interest-payment flows appears to have remained unexamined.

This note seeks to redress this gap on the basis of Foley's (1982, 1986) dynamic formalisation of Marx's (1885) Circuit of Capital. In its canonical formulation, the Circuit of Capital framework is purely dynamical in that all aggregate outlays and value flows are represented not with explicit reference to behavioural norms, but as dynamic, lagged functions of the past outlays and value flows that support them. As argued by dos Santos (2011), the resulting, explicitly dynamic account of the monetary circuits in capitalist reproduction offer a robust bases from which to consider the distinctive macroeconomic content of credit allocated to support productive capitalist undertakings and credit supporting consumption expenditures. As this note argues, the framework also allows a distinctive, Marxian approach to the class distribution of income: Wages engaging labour-power in production are a fraction of capital outlays, whose advance is prior and independent from the sale

[2] See also the recent contribution along the latter lines in Baragar and Chernomas (2012).

of the output commodities they eventually yield. Profits are the realisation through sale of mark-ups imbued onto output through the expropriation of unpaid labour time. Under this light, the wage and profit shares of income are not an *ex post* sharing of output. They are a dynamic result of independent capital-outlay and consumption decisions by different sectors in a capitalist economy.

On those two analytical bases it is possible to consider the distinctive dynamic and structural influences production and consumption credit have on the aggregate, class distribution of income.[3] This note does so by considering the comparative-dynamic impact of changes to each type of borrowing on income distribution across exponential steady-state evolutions. It establishes that in the Circuit of Capital, these two credit allocations have opposite dynamic effects on the aggregate distribution of income. Production credit immediately boosts wage flows. But time lags in consumption expenditures by workers and by capitalists ensure that, along a steady-state dynamic evolution, this lending makes a smaller contemporaneous contribution to commodity sales, and hence to profits. Conversely, consumption credit immediately boosts sales flows that realise profits. But inasmuch as the recommitment of retained earnings to capital outlays is subject to lags, its contemporaneous contribution to wage flows will be smaller. As a result production credit increases aggregate wages relative to profits, while consumption credit increases aggregate profits relative to wages.

While these results are established within the purely structural terms of the dynamic Circuit of Capital formalisation, the note also lays out the generally plausible aggregate behavioural norms governing outlays that that would ensure their broader purchase along steady-state evolutions. As such, the findings advanced by this note are not simply of analytical interest but also have potential policy significance. They should strike an additional note of caution regarding the ongoing policy enthusiasm for household credit across a range of advanced and middle-income economies as a tool for macroeconomic management and for facilitating private, market-based delivery of services in housing, health, and education. Not only are there theoretical reasons to expect it to pose endogenous tendencies towards credit-market instability,[4] but it may additionally make an independent contribution to greater levels of inequality, through the interest-payments flows it creates, and by virtue of its very location in the dynamic flows of value in capitalist reproduction.

The note continues as follows. Section two lays out the formal framework, including the Circuit of Capital and its corollary appreciation of the class distribution of income. Section three provides the note's principal discussions, offering new tools facilitating comparative-dynamic analysis of demand-determined, general versions of the Circuit of Capital--*demand shares* and the *effective*

[3] This implies abstracting from explicit consideration of mortgage borrowing, and the possible impact of house-price changes on household expenditures and income distribution. A brief discussion of how the present discussion does and does not address this important type of borrowing is offered in the conclusion.

[4] as formally argued in dos Santos (2011, 2013).

velocity of money. The section also offers comparative-dynamic analyses of exponential steady-state evolutions for the economy outlined in section two; and discusses the general purchase of the note's findings beyond the Circuit of Capital formalisation. Section four concludes.

2. The Formal Framework

This note deploys the most general formulation of the Circuit of Capital, laid out by Foley (1986), with a general parametrisation of net borrowing as leverage ratios boosting self-financed outlays by capitalist enterprises and wage-earning households. The Circuit of Capital offers a purely dynamic, integrated approach to production and exchange in a two-class capitalist economy, founded on the reproduction schema of Marx (1885). It considers the process of accumulation in a setting in which neither labour nor other resource constraints limit economic growth. In such a case, the only supply constraint bearing on accumulation is the dynamic, time productivity of labour, which limits the paces at which inputs engaged by capital outlays are transformed into outputs available for sale. Accumulation is also understood to be constrained by its endogenous ability to sustain demand allowing commodity sales that realise mark-ups imbued into them during production.

The Circuit of Capital exhibits two additional defining features. First, it is a structural, underdetermined stock-flow-consistent framework, in which capital value is taken to flow through three different phases: as money or financial assets held by capitalist enterprises, as engaged labour-power and non-labour inputs, and as finished but unsold output commodities. The flows of value between each successive phase are understood in purely dynamic terms. Present outflows of value from each phase of the circuit are approached as functions of past inflows into that phase. Thus present capital outlays are understood in relation to past enterprise retained earnings; present output flows are understood in relation to past capital outlays; and present sales are considered in relation to inventories fed by past flows of output commodities.

Second, in the Circuit of Capital, aggregate class income flows are not understood as *ex post* shares of total output. In line with Marx's approach, wages are understood as a definite part of capital outlays. Their exact proportion in those outlays defines the *composition of capital*. Profits are understood as the monetary realisation, through the eventual sale of output commodities, of the mark-up contained in those commodities. This mark-up is understood as a result of the *exploitation* of labour power during production; that is, the fact that workers only receive payment for a fraction of their labour time. The payment of wages to mobilise labour power in commodity production logically and temporally precedes the purchase of produced output commodities. They each follow from distinct outlay decisions by different sectors of the economy. As a result, the relative measure of wages and profits is most usefully understood endogenously and dynamically.

This section lays out a general circuit-of-capital framework that permits comparative examination of the relationship between credit allocation and the aggregate, class distribution of income.

2.1 Flows of Value

The Circuit of Capital commences with the advance of value as capital outlays by enterprises, which mobilise labour power and non-labour inputs to start production. The proportion of wage expenditures in total capital outlays, or composition of capital, is denoted by κ_t, ensuring that total outlays Z_t are decomposed as,

$$Z_t = \kappa_t Z_t + \left(1 - \kappa_t\right) Z_t \tag{1}$$

The emergence of output commodities from an original advance of capital value is understood to proceed dynamically, according to the time-productivity of labour. This is made operational with the use of a lag function $x_p(t-\tau;t)$, which describes the proportions of the value previously committed as capital outlays at times $\tau \le t$ that are emerging as finished output commodities at time t. The preservation of value requires that,

$$\int_{\tau}^{\infty} x_p(t-\tau;t)\,dt = 1, \tag{2}$$

ensuring all capital value committed to production eventually emerges as output commodities. The present flow of output may thus be expressed as,

$$P_t = \int_{-\infty}^{t} Z_\tau x_p(t-\tau;t)\,d\tau \tag{3}$$

The sale of output is taken to realise a mark-up on input costs, measured by q_t. In line with Marxian approaches, the mark-up is understood as a product of the composition of capital and the *rate of exploitation*, ε_t, or ratio between unpaid and paid labour time in the process of production,

$$q_t = \varepsilon_t \kappa_t \tag{4}$$

Demand flows D_t will finance sales that realise mark-ups on the costs R_t of commodities sold, so that, $D_t = \left(1+q_t\right)R_t$.[5] Sales are supported by three demand flows: demand by capitalist enterprises for non-labour inputs, and demand by workers and capitalists for consumption goods.

Workers are taken to spend their wage income dynamically, in line with a lag process describing the proportion of their past income currently used to purchase goods. If this lag process is denoted by $x_w(t-\tau;t)$, their present, self-financed consumption expenditure D_t^O will be given by,

$$D_t^O = \int_{-\infty}^{t} \kappa_\tau Z_\tau x_w(t-\tau;t)\,d\tau \tag{5}$$

Workers are also taken to support a fraction υ_t of their total consumption D_t^w through net borrowing, ensuring their total demand is given by,

$$D_t^w = \frac{1}{\left(1-\upsilon_t\right)} \int_{-\infty}^{t} \kappa_\tau Z_\tau x_w(t-\tau;t)\,d\tau \tag{6}$$

Capitalist households receive a fraction $\left(1-p_t\right)$ of total profits presently realised as income. Their expenditure is also subject to a lag process, in this case, $x_c(t-\tau;t)$. Assuming they do not borrow, aggregate capitalist demand for consumption goods will be given by,

$$D_t^c = \int_{-\infty}^{t} (1-p_\tau)q_\tau R_\tau x_c(t-\tau;t)\,d\tau \tag{7}$$

Capitalist enterprises retain a fraction p of profits plus the costs recovered through sales of output. Their reinvestment of retained own funds is taken to be subject to a lag process, $x_v(t-\tau;t)$. Assuming a fraction λ_t of capital outlays is supported through net borrowing by enterprises, total capital outlays will be given by,

$$Z_t = \frac{1}{(1-\lambda_t)} \int_{-\infty}^{t} (1+p_\tau q_\tau)R_\tau x_v(t-\tau;t)\,d\tau \tag{8}$$

Aggregate demand will be the sum of (6), (7), and a fraction $\left(1-\kappa\right)$ of present capital outlays,

[5] Note the assumption here that commodity sales realise present mark-ups.

$$D_t = (1+q_t)R_t = (1-\kappa_t)Z_t + D_t^w + D_t^c \tag{9}$$

This can be expressed as an explicit account of the endogenous sources of present demand flows,

$$(1+q_t)R_t = (1-\kappa_t)Z_t + \frac{1}{(1-\upsilon_t)}\int_{-\infty}^{t}\kappa_\tau Z_\tau x_w(t-\tau;t)\,d\tau + \int_{-\infty}^{t}(1-p_\tau)q_\tau R_\tau x_c(t-\tau;t)\,d\tau \tag{10}$$

Equations (8) and (10) summarise aggregate developments in the sphere of circulation of the economy in question.

2.2 Stocks, Production Constraints, and Income Distribution

Three additional pieces of the Circuit of Capital are necessary to complete the framework used below. First, the circuit exhibits three of stocks of value evolving as a result of the flows detailed above: Money hoards of capitalist enterprises M_t^e, which evolve as earnings are retained and as self-financed capital outlays are undertaken; productive capital as yet to complete its transformation into output Π_t, which evolves as capital outlays engage more inputs and as finished output emerges; and inventories N_t, which, accounting at costs, evolve as output is produced and costs are recovered through sales. Formally,

$$\dot{M}_t^e = (1+p_t q_t)R_t - (1-\lambda_t)Z_t \tag{11}$$

$$\dot{\Pi}_t = Z_t - P_t \tag{12}$$

$$\dot{N}_t = P_t - R_t \tag{13}$$

Total capital value engaged by capitalist enterprises will be given by the sum of these three stocks, and evolves in line with $\dot{K}_t = p_t q_t R_t + \lambda_t Z_t$. The unspent wage and profit revenues of wage-earning and capitalist households define their respective circuits of revenue. As only money is held as an asset in this setting, the money holdings of each of these sectors evolves in line with,

$$\dot{M}_t^w = \kappa_t Z_t - (1-\upsilon_t)D_t^w \tag{14}$$

$$\dot{M}_t^c = (1-p_t)q_t R_t - D_t^c \tag{15}$$

Since total net borrowing is given by $\dot{B}_t = \upsilon_t D_t^w + \lambda_t Z_t$, (9), (11), (14) and (15) ensure this borrowing equals the total change in money holdings $\dot{M}_t = \dot{M}_t^e + \dot{M}_t^w + \dot{M}_t^c = \dot{B}_t$.

Second, the productive or supply constraints bearing on accumulation are defined in the present setting by the evolution of inventories in (13). Demand flows can propel accumulation no further than inventories ensure there are output commodities available for sale. Along any given evolution, real growth requires capital outlays must support the production of commodities at dynamic paces sufficient to ensure inventories remain positive, given demand flows.

Third and finally, the Circuit of Capital's dynamic framework affords a fresh perspective on the class distribution of income. Considering this distribution as the ratio of profit to wage flows, and using the Marxian definition of the mark-up rate in (4), yields,[6]

$$H_t \equiv \frac{q_t R_t}{\kappa_t Z_t} = \varepsilon_t \frac{R_t}{Z_t} \qquad (16)$$

Equation (16) provides a new, Marxian appreciation of the concrete, realised expression of the distribution of money income in a capitalist economy. In line with extant Marxian interpretations, this distribution appears as conditioned by the rate of exploitation, understood to be defined in the sphere of production and in the broader struggles between labour and capital. But it also appears as conditioned by the relative aggregate measure of revenues to outlays of capitalist enterprises, which is defined by a range of spending decisions in the sphere of commodity circulation. The realised class distribution of income may be understood as the concrete realisation of the more general or essential rate of exploitation, as mediated by processes in circulation that condition the paces at which produced commodities are purchased, relative to capital outlays of enterprises.

This constitutes a significantly different appreciation of income distribution than under neoclassical formalisations, where perfectly competitive markets ensure factor shares are understood to results from "technology", and to be given by the corresponding factor elasticity of output in the aggregate or "representative" production function. It is also very different from conventional appreciations of the distribution of income as an *ex post* sharing of output. Here the aggregate distribution of income is seen as a consequence of decisions to produce by capitalist enterprises, and decisions to consume by wage-earning and capitalist households. Inasmuch as the latter decisions are held not to follow automatically from the former over some time horizon, demand determination of output becomes analytically significant, and the aggregate distribution of income emerges as an endogenous consequence of separate sectoral expenditure decisions: those of capitalist enterprises (and possibly other sectors, like government) to engage in capital outlays, those of capitalist and workers' households (and possibly others) to consume. It also hinges on the rate of exploitation, which may itself evolve as the pace of capital outlays tightens and loosens labour markets.

[6] Note that it is implied here that sales realise reproduction labour times, not production labour times.

Even simple dynamic dependences between the three terms in (16) can yield complex evolutions and dynamic relationships for income distribution. Approaching distribution as either technologically given, or as an exogenous parameter will in this context amount to the imposition of strong constraints on related but nevertheless independent sectoral capital and consumption outlay decisions, and on the evolution of the rate of exploitation.

The view taken here is that it is analytically more fruitful to consider it as an endogenous outcome of sectoral spending decisions and the outcome of market and broader struggles between labour and capital. While not pursued here, if distribution is taken as an endogenous variable, great care is needed when inquiring into its relationship with other endogenous variables, like the economy's rate of growth. More germanely to present purposes, the impact of any exogenous parameter on the aggregate, class distribution of income will hinge on its relative impact on capital outlays, aggregate demand, and the rate of exploitation. Along these lines, the next section considers the impact of changes in paces of different types of credit on aggregate income distribution across exponential, steady-state evolutions in the circuit-of-capital framework laid out above.

3. Exponential Steady States and Their Comparative Properties

Equations (3), (6) - (8), and (10) - (15) describe the dynamic evolution of an economy with five flows and five stocks. Together with the production or supply constraint requiring that inventories always be positive, they yield a well-specified system on the economy's stocks and flows of value. As long as the supply constraint is satisfied, this economy's dynamic evolution will be conditioned entirely in the sphere of circulation. This section finds exponential steady-state solutions to this system in such situations, and uses the results to establish the comparative dynamic relationship between different credit allocations and the distribution of income along those evolutions. In so doing, the section lays out and motivates this note's principal contributions and arguments.

3.1 Exponential Steady States and Endogenous Determinants of Growth

If the leverage ratios, the composition of capital, the mark-up rate, and rate of profit capitalisation are taken as exogenous parameters, and the expenditure lag processes do not change over time, equations (8) and (10) are two Volterra integral equations with difference Kernels on Z_t and R_t.

They summarise the relationship in the sphere of exchange between aggregate demand and the realisation of mark-ups through the sale of output commodities. As long as supply constraints are non-binding, these two equations constitute a subsystem whose solutions define the solutions for all stocks and flows in the economy. Systems like those posed by (8) and (10) admit exponential solutions, which will depend on the Laplace Transformations of their Kernels, in this case the lag processes $x_i(t)$, taken for the system's rate of growth g,

$$x_i^*(g) \equiv \mathcal{L}\{x_i(t), g\} = \int_0^t e^{-gt} x_i(t)\,dt \le 1 \tag{17}$$

Along exponential steady-state evolutions for all stocks and flows in the economy, (8) and (10) respectively become,[7]

$$Z_t = \frac{(1+pq)}{(1-\lambda)} R_t x_v^*(g) \tag{18}$$

$$(1+q)R_t = (1-\kappa)Z_t + \frac{\kappa}{(1-\upsilon)} x_w^*(g) Z_t + (1-p)qR_t x_c^*(g) \tag{19}$$

Substituting (18) into (19) yields the characteristic equation for this system,

$$(1+q)R_t = (1-\kappa)\frac{(1+pq)}{(1-\lambda)} x_v^*(g) R_t + \kappa \frac{(1+pq)}{(1-\upsilon)(1-\lambda)} x_w^*(g) x_v^*(g) R_t + (1-p)q x_c^*(g) R_t \tag{20}$$

Equation (20) summarises the relationship between commodity output values being presently realised through sales and the three components of aggregate demand; all expressed in relation to the production cost of commodities being sold. It may be more elegantly stated by defining the respective *demand shares* represented by different sectoral expenditures: enterprise outlays on non-labour inputs or means of production, and consumption outlays by workers and capitalists,

$$\rho_{mp}[\vec{x}, g] \equiv (1-\kappa)\frac{(1+pq)}{(1-\lambda)} x_v^*(g) \tag{21}$$

$$\rho_w[\vec{x}, g] \equiv \kappa \frac{(1+pq)}{(1-\lambda)(1-\upsilon)} x_v^*(g) x_w^*(g) \tag{22}$$

$$\rho_c[\vec{x}, g] \equiv (1-p) q x_c^*(g) \tag{23}$$

Where $\vec{x}$ is the vector of all parameters in the framework.[8] Define also $\rho_s[\vec{x}, g] \equiv -(1+q)$, the supply "share" of demand. According to these, (20) becomes,

$$F[\vec{x}, g] = -\rho_s[\vec{x}, g] - \rho_c[\vec{x}, g] - \rho_w[\vec{x}, g] - \rho_{mp}[\vec{x}, g] = 0 \tag{24}$$

$F[\vec{x}, g]$ may be understood as an "excess supply" function in exponential steady-states, along which it is, by definition, zero. As long as production constraints are non-binding, this function may be taken to define the necessary steady-state relationship between the system's parameters and its

[7] See Polyanin and Manzhirov (2008), p 115.

[8] Note that the lag functions are continua of parameters.

endogenous rate of growth, $g = g[\vec{x}]$. While it is not possible to find an explicit solution for this rate of growth without specifying functional forms for the lag functions, two properties of the Laplace Transform help provide insight into the underlying relationships.

First, by (17), $x_i^*(0) = 1, \forall i$. Second, since $x_i^*(t) \geq 0$, $x_i^*(g)$ is monotonically negative on g,

$$\frac{d}{dg} x_i^*(g) = \frac{d}{dg} \int_0^t e^{-gt} x_i(t) dt = -\int_0^t t e^{-gt} x_i(t) dt = -\mathcal{L}\{t x_i(t), g\} \leq 0 \tag{25}$$

If no net borrowing takes place, or $\upsilon = \lambda = 0$, (20) will hold when $g = 0$. But since all the terms $x_i^*(g)$ in the right-hand side of that equation are decreasing on g, that is its only solution. If the rate of growth is greater than zero, it is clear that positive net borrowing is needed to ensure (20) holds along a steady state with constant lag functions.

The comparative-dynamic relationship between any two variables χ and ϑ in (20) or (24) will be given, via the Implicit Function Theorem,

$$\frac{d\chi}{d\vartheta} = -\frac{F_\vartheta[\vec{x}, g]}{F_\chi[\vec{x}, g]} \tag{26}$$

The relationship between the parameters in $\vec{x}$ and the system's steady-state rate of growth is of particular interest. It will always involve $F_g[\vec{x}, g]$, which as shown in Appendix B is given by,

$$F_g[\vec{x}, g] \equiv \rho_c[\vec{x}, g] \frac{\mathcal{L}\{t x_c(t), g\}}{x_c^*(g)} + \rho_w[\vec{x}, g] \left(\frac{\mathcal{L}\{t x_w(t), g\}}{x_w^*(g)} + \frac{\mathcal{L}\{t x_v(t), g\}}{x_v^*(g)} \right) + \rho_{mp}[\vec{x}, g] \frac{\mathcal{L}\{t x_v(t), g\}}{x_v^*(g)} \tag{27}$$

The terms containing ratios of Laplace Transforms in (27) provide measures of the time delays involved in the passage of value through the respective phase of the circuit of capital. In the numerator, the Laplace Transform provides a present-discounted measure of the average length of time taken by a pulse of value to traverse the phase in question. The denominator normalises this to a measure of the discounting factor being applied, resulting in a dynamic measure of delay,[9]

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[9] In fact, for discrete delays of size d_i, represented by $x_i(t) = \delta(d_i)$, the Dirac Delta function, the average delay Δ_i will be exactly equal to the time delay d_i. For exponential decays of the form $x_i(t) = ae^{-at}$, the average delay will be given by $(a+g)^{-1}$.

$$\Delta_i\left[g,x_i(t)\right] \equiv \frac{\mathcal{L}\left\{tx_i(t),g\right\}}{x_i^*(g)} \tag{28}$$

This ensures (27) becomes,

$$F_g[\vec{x},g] = \rho_c[\vec{x},g]\Delta_c + \rho_w[\vec{x},g]\left(\Delta_w + \Delta_v\right) + \rho_{mp}[\vec{x},g]\Delta_v \tag{29}$$

Figure 1 helps provide an intuitive interpretation for the partial derivative in (28). Is the sum of all three demand shares, each multiplied by a measure of the delays involved in the movement of money from past sales to present demand. It may be understood as a specific, leverage-adjusted measure of the average delay involved in the movement of money from sale to sale. It is leverage-adjusted in that it actually measures the average delay in a hypothetical setting in which all demand is financed from own funds.

A related interpretation of the measure arising from (29) is through it's reciprocal quantity, $\mathcal{V}[\vec{x},g] \equiv F_g[\vec{x},g]^{-1}$. It provides a measure of the average, *effective* velocity of money along the steady state. The measure is "effective" because it accounts not only for the average quickness with which money reappears as demand following a sale, but also for the fact that along some channels of circulation it reappears boosted or leveraged by new money created through net credit extension.

<FIGURE 1 Somewhere Around Here>

From equations (26) and (29) it is possible to characterise the steady-state, comparative dynamic relationship between all parameters ζ and the system's rate of growth,

$$\frac{dg}{d\zeta} = \mathcal{V}[\vec{x},g]\sum_i \frac{\partial}{\partial\zeta}\rho_i[\vec{x},g] \tag{30}$$

Equation (30) offers a new and convenient shorthand for establishing the comparative-dynamic effect of a marginal parameter change on the real or trend rate of growth in a demand-constrained system, and validates the interpretation of the inverse of the derivative in (29) as a measure of the velocity of money. Such an effect will be given by the sum of the effects of the parameter change on the components of aggregate demand, multiplied by the average, effective or leverage-adjusted measure of the velocity of money. Once posed, this result is quite intuitive for any dynamic, steady-state evolution in which the rate of growth is demand determined.

Equation (30) yields the two comparative-dynamic derivatives of interest to this note,

$$\frac{dg}{d\lambda} = \frac{1}{(1-\lambda)} \mathcal{V}[\vec{x},g]\left\{\rho_{mp}[\vec{x},g] + \rho_w[\vec{x},g]\right\} \tag{31}$$

$$\frac{dg}{d\upsilon} = \frac{1}{(1-\upsilon)} \mathcal{V}[\vec{x},g]\rho_w[\vec{x},g] \tag{32}$$

Four intuitive observations follow from these two results. First, these derivatives are trivially positive since the demand shares and average delays are all positive, and the leverage ratios are only defined over the interval (-1,1) over which the hyperbolic terms in (31) and (32) are positive. This is not surprising, as growth in an economy whose production or supply constraints are non-binding will be determined by demand which, *ceteris paribus*, is boosted by higher leverage ratios.

Second, the capacity of each type of credit to boost growth is proportional to the total demand share of the expenditures it supports directly. Production credit affects growth in line with the proportion of capitalist demand for means of production and workers' demand for consumption goods in aggregate demand. Consumption credit boosts growth in line with the proportion of total demand accounted by workers' consumption. Third, and intuitively, the ability of both types of credit to boost the economy's rate of growth is proportional to the effective velocity of money, which measures the pace at which the newly created money will appear as demand for commodities. Finally and fourth, the hyperbolic terms in (31) and (32) appear as a result of the credit accelerator inbuilt in the parametrisation offered in section 2. Leverage boosts outlays, which in turn boost enterprise and workers' revenues, which boost self-financed outlays, boosting the contribution of any given leverage ratios to demand, and so on.

3.2 Comparative Dynamics on Distribution and their General Purchase

The class distribution of income, as measured by (16), will, along exponential steady states, be given by (18), which yields,

$$H[\vec{x},g] = \frac{q}{\kappa} \frac{(1-\lambda)}{(1+pq)x_v^*(g)} \tag{33}$$

The class distribution of income is taken here as an endogenous functions of the framework's parameters. It is also appears as a function of the endogenous rate of growth, which is itself endogenous. In such a setting, it makes no sense to speak of the impact of changes in distribution on growth. Both variables may change across steady states as particular movements in parameter space are considered. Their relative movements in response to a given parameter change can be characterised using (30) and (33). The implications of these observations for many contributions to the extant and growing literature on "wage-led" and "profit-led" growth are significant and require (forthcoming) separate discussion.

For now consider the simpler task of identifying the impact of changes in each of the two net borrowing parameters on the class distribution of income. Specifically, the total derivative of the income distribution function with respect to each of these parameters will be given by,

$$\frac{d}{d\chi}H[\vec{x},g]=H_{\chi}[\vec{x},g]+H_{g}[\vec{x},g]\frac{dg}{d\chi} \tag{34}$$

To identify this total derivative for the two net borrowing parameters, it is necessary, in addition to (31) and (33), to compute the partial derivatives with respect to g, λ, and υ of $H[\vec{x},g]$. These are,

$$H_{g}[\vec{x},g]=\frac{q(1-\lambda)}{\kappa(1+pq)}\frac{\mathcal{L}\{tx_{v}(t),g\}}{x_{v}^{*}(g)^{2}}\geq 0 \tag{35}$$

$$H_{\lambda}[\vec{x},g]=-\frac{q}{\kappa(1+pq)x_{v}^{*}(g)}\leq 0 \tag{36}$$

$$H_{\upsilon}[\vec{x},g]=0 \tag{37}$$

Substitution of equations (32), (35), and (37) into (34) will yield, after some manipulation, the expression for the comparative-dynamic consumption-leverage elasticity of income distribution,

$$\varepsilon_{\upsilon}^{H}\equiv\frac{dH}{d\upsilon}\frac{\upsilon}{H}=\frac{\upsilon}{(1-\upsilon)}\mathcal{V}[\vec{x},g]\rho_{w}[\vec{x},g]\Delta_{v}\geq 0 \tag{38}$$

Along the same lines, the elasticity of income distribution relative to leverage in capital outlays across comparative steady states may be obtained,

$$\varepsilon_{\lambda}^{H}\equiv\frac{dH}{d\lambda}\frac{\lambda}{H}=-\frac{\lambda}{(1-\lambda)}\mathcal{V}[\vec{x},g]\left\{\rho_{c}[\vec{x},g]\Delta_{c}+\rho_{w}[\vec{x},g]\Delta_{w}\right\}\leq 0 \tag{39}$$

These two comparative-dynamic results complement (31) and (32) and are the central formal findings advanced in this note. Both types of lending boost steady-state growth when production constraints are non-binding. But they do so while making opposite contributions to the distribution of income. Consumption credit increases the share of profits in total income, while production credit increases the share of wages.

The mechanisms responsible for these results in the present framework merit explicit discussion. Note first that the effects in (38) and (39) hinge respectively on the presence of lags in capital and consumption outlays, suggesting they are both fundamentally dynamic in the Circuit of Capital. The effects follow as a result of the positive impact all increases in leverage have on the rate of growth, and of the fact that production and consumption credit inject means of purchase into different locations along the dynamic sequence of monetary flows in capitalist reproduction. Along steady-state exponential evolutions, greater rates of growth increase the significance of any given lag structure in the movement of value. They will ensure present flows are larger relative to past flows and relative to stocks (which accumulate as functions of past flows). This in turn ensures that under higher growth rates, present *inflows* into any phase of the circuit are larger than present *outflows*, which are supported by past inflows in line with the relevant lag process. Since production and consumption credit directly support capital outlays and aggregate demand respectively, this ensures they have different comparative-dynamic effects on the relative measure of these two flows and, thus, different effects on the aggregate distribution of income.

In the present, demand-constrained setting, greater leverage ratios in capital outlays will, *ceteris paribus*, yield two comparative-dynamic results. They will increase the rate of growth and directly increase wage flows. The subsequent impact of greater wage flows on sales and profits is mediated by consumption lags. As these become more significant under the resulting higher rates of growth, consumption expenditure outflows, aggregate demand, sales, and profits will increase by a smaller proportion than capital-outlay and wage flows. As a (dynamic) result, the class distribution of income shifts in favour of wages. This effect may in fact be gathered directly from (19), after solving it for capital outlays,

$$R_t = \frac{(1-\kappa)+(1-\upsilon)^{-1}\kappa x_w^*(g)}{(1+q)-(1-p)qx_c^*(g)} Z_t \tag{40}$$

An increase in the dynamic pace of net extension of production credit will directly boost capital outlays by a given proportion. But the resulting increase in the rate of growth reduces the magnitude of the Laplace multipliers in (40), ensuring the increase in the level of sales, and thus profits, is proportionately smaller, resulting in improvements in the aggregate wage share. Greater leverage in consumption outlays by wage earners also results in steady states with higher rates of growth. But it directly increases sales and profit flows. Its full, dynamic effect on capital outlays is subject to the investment lag process. Applying the same reasoning and dynamic effects as above to equation (18) it is evident that the increase in capital outlays resulting from greater levels of net extension of consumption credit will be proportionately smaller than the corresponding increase in sales and profits.

While these effects have been established for steady-state evolutions defined within the abstract terms of a general, demand-constrained version of the Circuit of Capital, it is possible to motivate and characterise their purchase for more general evolutions. This includes those defined by explicit behavioural specifications of capital outlays and consumption expenditures. Along any such paths, the broad findings identified by this note will hold as long as for all sectors of the economy the elasticity of outlays relative to changes in revenues measures less than unity. This may be ascertained from equation (16), which makes clear that as long as the rate of exploitation is constant, the comparative impact of each type of credit on the distribution of income will hinge on the relative responsiveness of aggregate-demand and of capital-outlay flows to changes in the levels of each type of lending.

Since production credit directly affects capital outlays, its impact on distribution hinges on the responsiveness of aggregate demand to changes in capital outlays. Specifically, the result that this type of lending improves wage shares will hold as long as the elasticity of aggregate demand with respect to capital outlays is below unity. It is evident from accounting identity (9) that this will be the case as long as the respective income elasticities of demand of wage-earning and capitalist households are themselves less than one. Along analogous lines, the impact of consumption credit on income shares will hinge on the elasticity of capital outlays relative to changes in aggregate demand. The finding that this type of lending improves profit shares will hold as long as this elasticity is below unity, ensuring it is associated with stronger proportional effects on demand, sales, and profits, than on capital outlays and wage flows.

The types of aggregate-outlay behaviour that generally ensure the purchase of the results contained in (38) and (39) are plausible, especially over longer-term or trend evolutions. The requirement that the aggregate income elasticity of demand by wage-earning and capitalist households be below unity amounts to the simple supposition that sectoral marginal propensities to consume are falling on relative measures of aggregate wage and profit incomes.[10] As for the second requirement, it is entirely possible that over short periods capital outlays are disproportionately responsive to changes in relative measures of aggregate demand and sales. But any persistence in such aggregate behaviour would generate results at variance with the widely acknowledged and observed counter-cyclical movement in the relative measure of inventories of unsold commodities. In fact, a persistent elasticity of capital outlays relative to changes in demand in excess of unity would ensure that inventories *increase* in their relative measure as the pace of demand quickens, and *decrease* as demand slackens. Similarly, a persistent elasticity value of unity would ensure no movements in the relative measure of inventories, effectively ensure the absence of unplanned inventory movements, and amount to a *sui generis*, dynamic corroboration of Say's Law. Finally it should be noted that along any stable evolution at least one of the results in (38) and (39) will hold. If both the elasticity of capital outlays with respect to aggregate demand, and the elasticity of aggregate demand with respect to capital outlays exceed unity, the economy's evolution will be ex/implosive.

[10] It is easy to conceive of settings in which greater aggregate wage flows may be accompanied by disproportionately large consumption outlays, such as an expansion in capital outlays that relies on the integration into production of new workers with higher-than-average consumption propensities. But even if this expansion of employment is supported by greater enterprise borrowing, this would not be a *comparative dynamic* change, as the changes in the composition of the labour force are additionally chainging the average dynamic spending behaviour of workers. The resulting effects reflect properties not of credit flows but of labour markets.

4. Conclusions

This brief note has explicitly tackled the structural relationship between credit allocation and the distribution of income in a purely dynamic, Marxian analytical framework of accumulation. In line with that framework, wages were taken as a fraction of capital outlays, and profits as a fraction of sales of commodity output to capitalist enterprises, workers, and capitalist consumers. This resulted in a new, distinctive view of the aggregate, class distribution of income, which appears neither as an *ex post* sharing of output, nor as primarily defined by "technology". Instead, distribution was distinctively shown to emerge as an endogenous consequence of developments in the sphere of production and the sphere of exchange. In the former, it was shown to be conditioned, along conventional Marxian lines, on the rate of exploitation, given by the division of the average working day between paid and unpaid labour time. In the latter, distribution was shown also to be conditioned by the articulation of aggregate decisions to produce, which sustain capital outlays, and aggregate decisions to consume, which help determine aggregate demand.

On those bases, the note established that while credit allocated to production and credit allocated to consumption both dynamically boost steady-state rates of growth, they have fundamentally different impacts on the class distribution of income. While greater levels of production credit were shown to result in steady-state evolutions with higher wage shares in total income, greater levels of consumption credit were shown to yield economies with higher profit shares. While developed in the mathematically taxing, dynamic terms of a general-lag-process version of the Circuit of Capital, the note identified the conditions under which these findings will have more general purchase. Specifically, along any evolution in which the sectoral revenue elasticity of all outlays is less than unity, production credit will boost wage shares while consumption credit boosts profit shares.

These findings represent a new contribution to the understanding of the relationship between income distribution and credit allocation. They should also strike an additional note of caution regarding the recent policy enthusiasm for household indebtedness across advanced and middle-income economies, where it has been encouraged as a tool for macroeconomic demand management and to facilitate the market-based, private provision of social services in housing, health, and education.[11] Rising income inequality has been motivated as an important determinant of rising levels of household indebtedness in the US by contributions drawing form a range of traditions in economic analysis. Inasmuch as we may identify differences in the general distribution of household debt and in that of asset holdings that sustain that debt, in that they are respectively clustered at lower and higher ends of income scales, interest-payment flows will

[11] See dos Santos (2011, 2013), which also point to the endogenous tendencies towards credit-market instability created by this type of lending. Compared to production credit, this lending boosts sales relative to capital outlays, which boosts aggregate profitability, likely creating market-process signals encouraging its growth. At the same time, this type of lending makes a comparatively greater contribution to debt relative to aggregate income flows than production credit, making a greater contribution to potential credit crises.

ensure this lending effects important, regressive redistributions. The findings made in this note point to a further potential channel through which this type of lending may increase income inequality, in this case between labour and capital. This increase follows *structurally*, from the very location where consumption credit injects means of purchase into the dynamic flows of money in capitalist reproduction. It directly boosts sales, and thus profits, but fails to boost capital outlays, and thus wages, directly. In contrast, production credit boosts wage flows immediately, while only commensurately boosting sales, and thus profits, in the future.

The note also helped illustrate and modestly develop the distinctive analytical purchase of the Circuit of Capital formalisation of Foley (1982, 1986). Its central formal findings concerning credit allocation and distribution follow directly from the existence of dynamic lags in sectoral expenditures, illustrating their analytical significance in any considered approach to macroeconomy dynamics. In addition, the note offered new analytical concepts that facilitate consideration of the properties of demand-constrained evolutions in the most general versions of the Circuit of Capital: demand shares and the effective velocity of money.

Finally, this note points to two avenues for further work. First, it points to the need to consider explicitly mortgage credit, the consumption and balance-sheet considerations governing its demand by wage earners, and its impact on commodity demand and on distribution. The note's discussion has direct purchase on borrowing that finances the acquisition of new homes. It is the acquisition of existing homes that complicates matters considerably, posing the need for deliberate approaches to household asset holdings across housing and non-housing assets that are quite beyond the simple portfolios that can be considered on the present analytical bases. A few pertinent points may nevertheless be motivated on the present bases. These purchases may effect redistributions among wage earners, including with important generational components. And, inasmuch as greater levels of household borrowing sustain price appreciations that enable higher levels of consumption than otherwise--say, via wealth effects or withdrawals of home equity--this borrowing will boost profits and raise the possibility of effects on growth and the class distribution of income along the lines motivated here.

Second, the discussion above suggests the need for a critical engagement with existing discussions concerning "wage-led" and "profit-led" growth. This literature generally approaches aggregate income distribution as an exogenous, *ex-post* sharing of output. From the dynamic standpoint offered here, that analytical decision amounts to the imposition of a strong restriction on sectoral expenditures that bear no direct relationship to one another. This note has argued that if wages are understood as fractions of capital outlays, and profits as fractions of sales of commodities to capitalist enterprises, and to wage-earners and capitalist households, the best approach is to consider the aggregate distribution of income as an endogenous, emergent result of independent sectoral outlay decisions. If growth is also taken as endogenous, as is most often the case in endogenous growth macroeconomics, the relationship between growth and distribution requires deliberate analysis and will likely throw up counterintuitive results. In those cases, it will be necessary to consider how movements in the space of exogenous parameters affect growth and distribution, from which it may be possible to infer the relationship between the two endogenous variables along such parameter movements. That is clearly the subject of separate, forthcoming work, which, it is hoped, this note has helped motivate.

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Appendix A | Full Exponential Steady-State Solutions

Normalising the system to $R_0 = 1$

Flows

$$R_t = e^{gt} \tag{A1}$$

$$Z_t = \frac{(1+pq)}{(1-\lambda)} x_v^*(g) R_t \tag{A2}$$

$$D_t^w = \frac{\kappa(1+pq)}{(1-\lambda)(1-\upsilon)} x_v^*(g) x_w^*(g) R_t \tag{A3}$$

$$D_t^c = (1-p) q x_c^*(g) R_t \tag{A4}$$

$$P_t = \frac{(1+pq)}{(1-\lambda)} x_v^*(g) x_p^*(g) R_t \tag{A5}$$

Stocks

$$M_t^w = \frac{\kappa(1+pq)}{g(1-\lambda)} \left(1 - x_w^*(g)\right) R_t \tag{A6}$$

$$M_t^c = \frac{(1-p)q}{g} \left(1 - x_c^*(g)\right) R_t \tag{A7}$$

$$M_t^e = \frac{\left(1 - x_v^*(g)\right)(1+pq)}{g} R_t \tag{A8}$$

$$\Pi_t = \frac{\left(1 - x_p^*(g)\right)(1+pq) x_v^*(g)}{g(1-\lambda)} R_t \tag{A9}$$

$$N_t = \frac{(1+pq) x_v^*(g) x_p^*(g) (1-\lambda)^{-1} - 1}{g} R_t \tag{A10}$$

The Rate of Growth

Is defined by parameter vector $\vec{x}$, according to the system's characteristic equation,

$$F[\vec{x}, g] = (1+q) - \rho_c[\vec{x}, g] - \rho_w[\vec{x}, g] - \rho_{mp}[\vec{x}, g] = 0 \tag{A11}$$

Supply Constraint

These solutions will describe the economy as long as $\vec{x}$ ensures positive inventories,

$$\frac{(1+pq)}{(1-\lambda)} x_v^*(g) x_p^*(g) \geq 1 \qquad \text{(A10)}$$

Appendix B | Derivations

Equations (26) - (28)

Starting from the full expression of the steady-state excess supply function $F[\vec{x},g]$,

$$F[\vec{x},g]=\left(1+q\right)-\left(1-\kappa\right)\frac{(1+pq)}{(1-\lambda)}x_v^*(g)-\kappa\frac{(1+pq)}{\left(1-\upsilon\right)(1-\lambda)}x_w^*(g)x_v^*(g)-(1-p)qx_c^*(g) \quad \text{(B1)}$$

To find its derivative with respect to the rate of exponential growth, we use the well-known result concerning the derivative of a Laplace Transformation with respect to the Laplace factor,

$$\frac{d}{dg}x_i^*(g)=\frac{d}{dg}\mathcal{L}\left\{x_i(t),g\right\}=\frac{d}{dg}\int_0^t e^{-gt}x_i(t)dt=-\int_0^t te^{-gt}x_i(t)dt=-\mathcal{L}\left\{tx_i(t),g\right\} \quad \text{(B2)}$$

Applying this property to take the derivative of (B1) with respect to g yields,

$$F_g[\vec{x},g]=\left(1-p\right)q\mathcal{L}\left\{tx_c(t),g\right\}+\frac{\kappa\left(1+pq\right)}{\left(1-\lambda\right)\left(1-\upsilon\right)}x_v^*(g)\mathcal{L}\left\{tx_w(t),g\right\}+\frac{\left(1+pq\right)}{(1-\lambda)}\left[1+\kappa\left(\frac{x_w^*(g)}{(1-\upsilon)}-1\right)\right]\mathcal{L}\left\{tx_v(t),g\right\} \quad \text{(B3)}$$

Using the definitions of the demand shares ρ_i given in (21), (22) and (23) ,

$$\rho_{mp}\left[\vec{x},g\right]\equiv\left(1-\kappa\right)\frac{\left(1+pq\right)}{\left(1-\lambda\right)}x_v^*(g) \quad \text{(B4)}$$

$$\rho_w\left[\vec{x},g\right]\equiv\kappa\frac{\left(1+pq\right)}{\left(1-\lambda\right)\left(1-\upsilon\right)}x_v^*(g)x_w^*(g) \quad \text{(B5)}$$

$$\rho_c\left[\vec{x},g\right]\equiv\left(1-p\right)qx_c^*(g) \quad \text{(B6)}$$

This becomes,

$$F_g[\vec{x},g]=\rho_c[\vec{x},g]\frac{\mathcal{L}\left\{tx_c(t),g\right\}}{x_c^*(g)}+\rho_w[\vec{x},g]\left(\frac{\mathcal{L}\left\{tx_w(t),g\right\}}{x_w^*(g)}+\frac{\mathcal{L}\left\{tx_v(t),g\right\}}{x_v^*(g)}\right)+\rho_{mp}[\vec{x},g]\frac{\mathcal{L}\left\{tx_v(t),g\right\}}{x_v^*(g)} \quad \text{(B7)}$$

Applying the definition of average delays given in (27),

$$\Delta_i=\frac{\mathcal{L}\left\{tx_i(t),g\right\}}{x_i^*(g)} \quad \text{(B8)}$$

we get result (28),

$$F_g[\vec{x},g] = \rho_c[\vec{x},g]\Delta_c + \rho_w[\vec{x},g]\left(\Delta_w + \Delta_v\right) + \rho_{mp}[\vec{x},g]\Delta_v \tag{B9}$$

Results (31) and (32)

The comparative-dynamic derivatives of the rate of growth on the leverage ratios follow from (30),

$$\frac{dg}{d\zeta} = \mathcal{V}[\vec{x},g]\sum_i \frac{\partial}{\partial\zeta}\rho_i[\vec{x},g] \tag{B10}$$

For capital-outlay leverage λ, the sum in this equation will contain the derivatives of the demand shares (B4) and (B5), which are given by,

$$\frac{d}{d\lambda}\rho_{mp}\left[\vec{x},g\right] = (1-\kappa)\frac{(1+pq)}{(1-\lambda)^2}x_v^*(g) = \frac{\rho_{mp}\left[\vec{x},g\right]}{(1-\lambda)} \tag{B11}$$

$$\frac{d}{d\lambda}\rho_w\left[\vec{x},g\right] = \kappa\frac{(1+pq)}{(1-\lambda)^2(1-\upsilon)}x_v^*(g)x_w^*(g) = \frac{\rho_w\left[\vec{x},g\right]}{(1-\lambda)} \tag{B12}$$

From which it is trivial to see that (B9) yields,

$$\frac{dg}{d\lambda} = \frac{1}{(1-\lambda)}\mathcal{V}[\vec{x},g]\left\{\rho_{mp}[\vec{x},g] + \rho_w[\vec{x},g]\right\} \tag{B12}$$

Along similar lines, the parallel result for leverage in workers' consumption will follow from the demand-share derivative,

$$\frac{d}{d\upsilon}\rho_w\left[\vec{x},g\right] = \kappa\frac{(1+pq)}{(1-\lambda)(1-\upsilon)^2}x_v^*(g)x_w^*(g) = \frac{\rho_w\left[\vec{x},g\right]}{(1-\upsilon)} \tag{B13}$$

From which it can be seen that,

$$\frac{dg}{d\upsilon} = \frac{1}{(1-\upsilon)}\mathcal{V}[\vec{x},g]\rho_w[\vec{x},g] \tag{B14}$$

Equations (38) and (39)

The principal results motivated in the note follow from the direct application of the total derivative of aggregate income distribution, as given by (34),

$$\frac{d}{d\chi}H[\vec{x},g]=H_{\chi}[\vec{x},g]+H_{g}[\vec{x},g]\frac{dg}{d\chi} \tag{B15}$$

For capital-outlay leverage this becomes,

$$\frac{dH}{d\lambda}=H_{\lambda}[\vec{x},g]+H_{g}[\vec{x},g]\frac{dg}{d\lambda} \tag{B16}$$

Expressing as a comparative-dynamic elasticity,

$$\varepsilon_{\lambda}^{H}\equiv\frac{dH}{d\lambda}\frac{\lambda}{H}=\frac{\lambda}{H}\left[H_{\lambda}[\vec{x},g]+H_{g}[\vec{x},g]\frac{dg}{d\lambda}\right] \tag{B17}$$

To render this into a function of exogenous parameters, we use result (B12), as well as the identities involving the distribution of income H given in (33) and (35), (36), and (37),

$$H[\vec{x},g]=\frac{q}{\kappa}\frac{(1-\lambda)}{(1+pq)x_{v}^{*}(g)} \tag{B18}$$

$$H_{g}[\vec{x},g]=\frac{q(1-\lambda)}{\kappa(1+pq)}\frac{\mathcal{L}\{tx_{v}(t),g\}}{x_{v}^{*}(g)^{2}} \tag{B19}$$

$$H_{\lambda}[\vec{x},g]=-\frac{q}{\kappa(1+pq)x_{v}^{*}(g)} \tag{B20}$$

$$H_{\upsilon}[\vec{x},g]=0 \tag{B21}$$

All of which yields (B22),

$$\varepsilon_{\lambda}^{H}=\lambda\frac{\kappa(1+pq)x_{v}^{*}(g)}{q(1-\lambda)}\left[\left(-\frac{q}{\kappa(1+pq)x_{v}^{*}(g)}\right)-\left(\frac{q(1-\lambda)}{\kappa(1+pq)}\frac{\mathcal{L}\{tx_{v}(t),g\}}{x_{v}^{*}(g)^{2}}\right)\left(\frac{1}{(1-\lambda)}\mathcal{V}[\vec{x},g]\left\{\rho_{mp}[\vec{x},g]+\rho_{w}[\vec{x},g]\right\}\right)\right]$$

Basic cancellation of terms straddling quotients simplifies this expression considerably,

$$\varepsilon_{\lambda}^{H} = \frac{\lambda}{(1-\lambda)}\left[(-1) + \frac{\mathcal{L}\{tx_v(t),g\}}{x_v^*(g)}\mathcal{V}[\vec{x},g]\left\{\rho_{mp}[\vec{x},g] + \rho_w[\vec{x},g]\right\}\right] \tag{B23}$$

Substitution of the average delay expression in (B8) and factoring out the effective velocity of money function yields,

$$\varepsilon_{\lambda}^{H} = \frac{\lambda}{(1-\lambda)}\mathcal{V}[\vec{x},g]\left[\Delta_v\left\{\rho_{mp}[\vec{x},g] + \rho_w[\vec{x},g]\right\} - \frac{1}{\mathcal{V}[\vec{x},g]}\right] \tag{B24}$$

But the reciprocal of the effective velocity of money is simply,

$$\mathcal{V}[\vec{x},g]^{-1} = \rho_c[\vec{x},g]\Delta_c + \rho_w[\vec{x},g](\Delta_w + \Delta_v) + \rho_{mp}[\vec{x},g]\Delta_v \tag{B25}$$

Whose substitution into (B24) trivially (by addition) results in the first of the main results,

$$\varepsilon_{\lambda}^{H} \equiv \frac{dH}{d\lambda}\frac{\lambda}{H} = -\frac{\lambda}{(1-\lambda)}\mathcal{V}[\vec{x},g]\left\{\rho_c[\vec{x},g]\Delta_c + \rho_w[\vec{x},g]\Delta_w\right\} \tag{B26}$$

Which for the parameter values under consideration will always be nonpositive.

The equivalent derivation for the consumption leverage parameter starts from the expression,

$$\varepsilon_{\upsilon}^{H} \equiv \frac{dH}{d\upsilon}\frac{\upsilon}{H} = \frac{\upsilon}{H}\left[H_{\upsilon}[\vec{x},g] + H_g[\vec{x},g]\frac{dg}{d\upsilon}\right] \tag{B27}$$

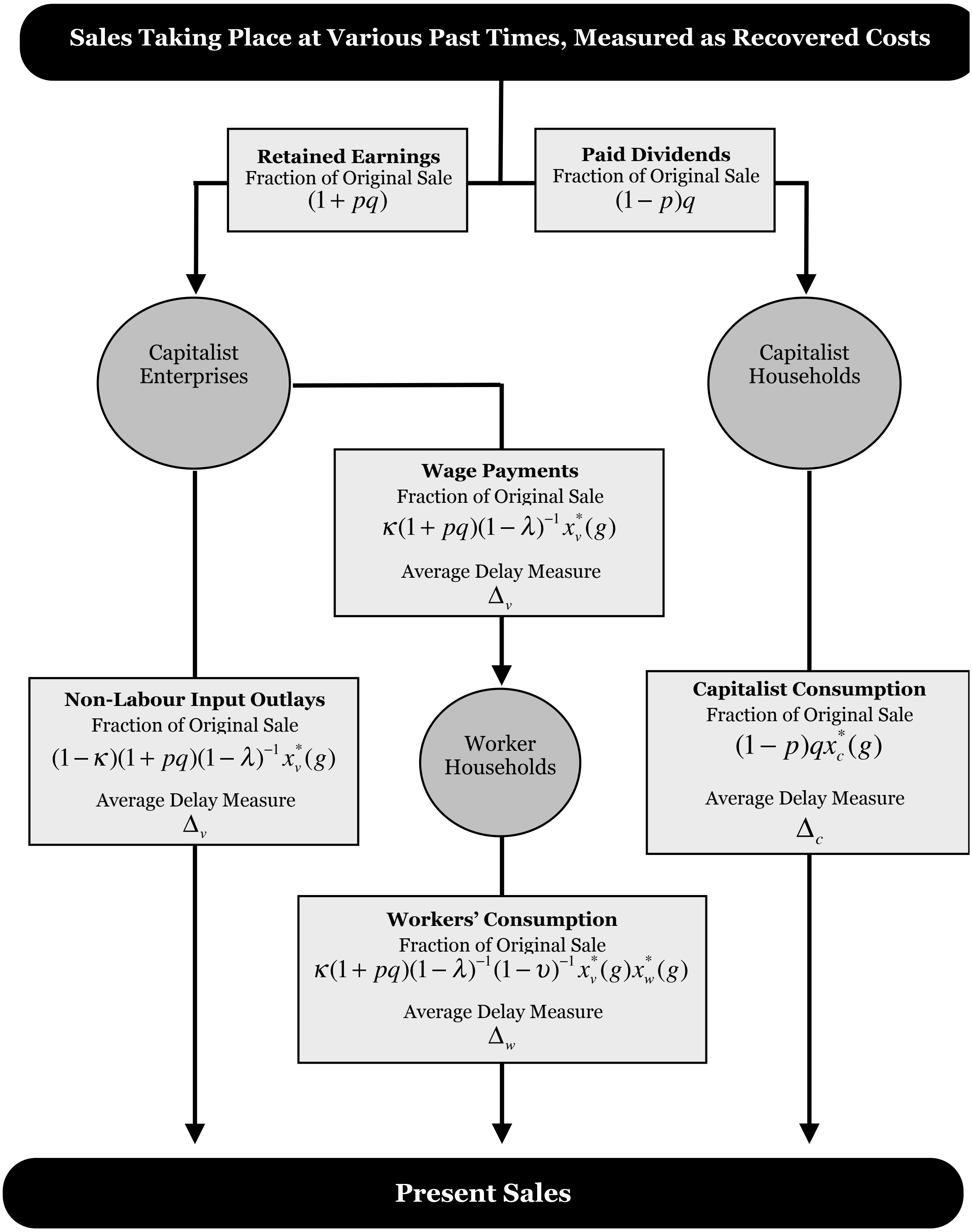
Substituting (B14), (B18), (B19), (B21) into this yields,

$$\varepsilon_{\upsilon}^{H} = \upsilon\frac{\kappa(1+pq)x_v^*(g)}{q(1-\lambda)}\left[\left(\frac{q(1-\lambda)}{\kappa(1+pq)}\frac{\mathcal{L}\{tx_v(t),g\}}{x_v^*(g)^2}\right)\left(\frac{1}{(1-\upsilon)}\mathcal{V}[\vec{x},g]\rho_w[\vec{x},g]\right)\right] \tag{B28}$$

Simple cancellation of terms, the substitution of the average-delay function, and rearrangement yield the final expression, which will always be positive over present parameter domains,

$$\varepsilon_{\upsilon}^{H} = \frac{\upsilon}{(1-\upsilon)}\mathcal{V}[\vec{x},g]\rho_w[\vec{x},g]\Delta_v \tag{B29}$$

Figure 1
Paths, Delays, and Relative Measures of Three Flows Supporting Present Demand



Demand Shares	$(1-\kappa)\dfrac{(1+pq)}{(1-\lambda)}x_v^*(g)$	$\kappa\dfrac{(1+pq)}{(1-\upsilon)(1-\lambda)}x_w^*(g)x_v^*(g)$	$(1-p)qx_c^*(g)$
Delay Shares	$\rho_{mp}[\vec{x},g]\Delta_v$	$\rho_w[\vec{x},g](\Delta_w+\Delta_v)$	$\rho_c[\vec{x},g]\Delta_c$